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An Enhanced Temimi-Ansari Method for Solving Nonlinear Fredholm Integro-Differential Equations

Nyikyaa M. N.[1], Kwami A. M.[2], A. G. Madaki[3], Okai J. O.[4]
Abubakar Tafawa Balewa University, Bauchi, Nigeria
jerry4complex@gmail.com

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Abstract

This study presents an enhanced version of the Temimi-Ansari Method (TAM) for effectively solving nonlinear integro-differential equations involving Fredholm-type integrals. The improved method builds upon the original TAM framework and demonstrates its robustness in addressing complex functional equations. Symbolic computation tools are employed to implement the method, and its performance is illustrated through several benchmark problems. The obtained results are compared with exact solutions and other semi-analytical techniques to validate the accuracy and efficiency of the proposed approach. The method proves to be computationally efficient, capable of simplifying calculations, and suitable for solving both linear and nonlinear Fredholm integro-differential equations of the second kind.

Keywords: Temimi-Ansari Method (TAM); Fredholm integro-differential equations; Semi-analytical techniques

INTRODUCTION

Integral equations are essential tools in both pure and applied mathematics, with widespread applications in science and engineering (Adwan & Rahdi, 2020; Okai, Kwami & Abubakar, 2020). According to Amin et al. (2021), these equations are particularly important in solving real-world problems in fields like engineering and mechanics. Integro-differential equations, which combine differential and integral terms, frequently appear in mathematical physics and engineering models (Arabia, 2021).

As noted by He and Virginia (2020), finding analytical solutions to these equations—especially when they are nonlinear—is often difficult. As a result, researchers have focused on developing reliable numerical and approximate methods to handle both linear and nonlinear forms. However, as Ijaiya, Taiwo & Bello (2021) and Okai, Manjak & Swem (2017) observed, nonlinear integro-differential equations are particularly challenging due to their complexity and practical significance.

Over the years, many numerical techniques have been introduced to address these equations, including the Galerkin method (Issa et al., 2022), Chebyshev and Taylor collocation methods (Mishra, 2017; Jafarzadeh & Keramati, 2018), rationalized Haar functions (Amin et al., 2021), Differential Transform Method (Ogunrinde, 2019), Adomian Decomposition Method (Tate & Dinde, 2019), Variational Iteration Method (Wazwaz, 2009), and several variants like the Laplace and Discrete Adomian Decomposition methods (Manafian, 2014; Bakodah & Almuhalbedi, 2019), as well as the New Iterative Method (Hemeda, 2015). These approaches aim to obtain accurate approximate solutions to nonlinear problems.

In 2011, Temimi and Ansari proposed a new iterative method (TAM) based on the Homotopy Analysis Method (HAM) (Shijun, 1998), designed for solving nonlinear problems without requiring linearization (Temimi & Ansari, 2011, 2015). TAM offers the benefit of generating analytic solutions while reducing computational effort. It has been successfully applied to various problems in science and engineering. However, TAM faces challenges when dealing with higher-order integro-differential equations involving complex nonlinearities, as it often leads to nested integrals that are difficult to evaluate analytically.

To address this limitation, this study proposes an improved method for solving nonlinear Fredholm integro-differential equations (NFIDEs). The approach combines the TAM with the Discrete Adomian Decomposition Method (Bakodah & Almuhalbedi, 2019) to effectively handle definite integrals and enhance the performance of TAM in solving complex equations.

METHODOLOGY

We propose the modified TAM to provide closed form solutions for nonlinear and linear Fredholm integro-differential equation; first we start with the introduction of the basic TAM then we propose the modification of the TAM for the solution of nonlinear and linear Fredholm integro-differential equation.

Temimi Ansari Method (TAM)

We introduce the following nonlinear differential equation:

$$L[u(x)] + N[u(x)] + g(x) = 0, \qquad ...(1)$$

with the boundary conditions

$$B\left[u, \frac{du}{dx}\right] = 0, \; x \in D \qquad ...(2)$$

where x represents the independent variable, $u(x)$ is the unknown function, $g(x)$ is a given known function, $L(.) = \frac{d^2}{dx^2}(.)$ is the linear operator, $N(.)$ is the nonlinear operator, $B(.)$ is a boundary operator. Now, let us begin by introducing the basic ideas of TAM. We first begin by assuming that $u_0(x)$ is an initial guess to solve the problem $u(x)$ and the solution begins by solving the following initial value problem [Temimi & Ansari, (2011)]:

$$L[u_0(x)] + g(x) = 0, \text{ and } B\left[u_0, \frac{du_0}{dx}\right] = 0 \qquad ...(3)$$

The next approximate solutions are obtained by solving the following problems

$$L[u_1(x)] + g(x) + N[u_0(x)] = 0, \text{ and } B\left[u_1, \frac{du_1}{dx}\right] = 0 \qquad ...(4)$$

and thus we have a simple iterative procedure which is the solution of a set of problems i.e.,

$$L[u_{n+1}(x)] + g(x) + N[u_n(x)] = 0, \text{ and } B\left[u_{n+1}, \frac{du_{n+1}}{dx}\right] = 0 \qquad ...(5)$$

Then, the solution for the problem (1) with the boundary conditions (2) is given by

$$u(x)=\lim_{n\to\infty}u_n(x) \quad ...(6)$$

We now illustrate the steps of the MTAM by considering the first-order FIDEs, second-order FIDEs, and third-order FIDEs.

Modifications of the Temimi Ansari Method for First-order FIDEs

Consider the nonlinear first-order Fredholm integro-differential equation:

$$u'(x)=f(x)+\lambda\int_0^1 k(x,t)N[u(t)]dt\ ,u(0)=u_0 \quad ...(7)$$

Applying the one-fold integral operator $L^{-1}(.)=\int_0^x (.)dt$ to both sides of Eqn. (7) and using the given initial condition, we obtain:

$$u(x)=u_0+\int_0^x f(t)dt+\lambda x\int_0^1 k(x,t)N[u(t)]dt \quad ...(8)$$

Eqn. (8) becomes

$$\sum_{n=0}^{\infty}u_n(x)=u_0+\int_0^x f(t)dt+\lambda x\int_0^1 k(x,t)N[u_n(t)]dt \quad ...(9)$$

Consequently, we set the following modified recursion scheme:

$$u_0(x)=f(x), \quad ...(10)$$

Where $f(x)=u_0+\int_0^1 f(t)dt$

$$u_1(x)=u_0(x)+\lambda x\int_0^1 N[u_0(t)]dt, \quad ...(11)$$

$$u_{n+1}(x)=u_0(x)+\lambda x\int_0^1 N[u_n(t)]dt,\ n\geq 1 \quad ...(12)$$

For the determination of the solution components $u_n(x)$, $n\geq 0.$ in view of this, the solution approximant $\phi_{m+1}(x)$ is given by:

$$\phi_{m+1}(x)=\sum_{n=0}^{m}u_n(x), \qquad ...(13)$$

That gives:

$$\lim_{m\to\infty}\phi_{m+1}(x)=\sum_{n=0}^{m}u_n(x)=u^*(x). \qquad ...(14)$$

Modifications of the Temimi Ansari Method for Second-order FIDEs

Consider the nonlinear second-order Fredholm integro-differential equation:

$$u''(x)=f(x)+\lambda\int_0^1 k(x,t)N[u(t)]dt\,,u(0)=c_0\,,u'(0)=c_1 \qquad ...(15)$$

Applying the two-fold integral operator $L^{-1}(.)=\int_0^x\int(.)dtdt$ to both sides of Eqn. (15) and using the given initial conditions, we obtain:

$$u(x)=c_0+c_1x+\int_0^x\int_0^x f(t)dtdt+\lambda\frac{x^2}{2!}\int_0^1 k(x,t)N[u(t)]dt \qquad ...(16)$$

Eqn. (16) becomes

$$\sum_{n=0}^{\infty}u_n(x)=c_0+c_1x+\int_0^x\int_0^x f(t)dtdt+\lambda\frac{x^2}{2!}\int_0^1 k(x,t)N[u_n(t)]dt \qquad ...(17)$$

Consequently, we set the following modified recursion scheme:

$$u_0(x)=f(x), \qquad ...(18)$$

Where $f(x)=c_0+c_1x+\int_0^x\int_0^x f(t)dtdt$

For the determination of the solution components $u_n(x)$, $n\geq 0.$ in view of this, the

$$u_1(x)=u_0(x)+\lambda\frac{x^2}{2!}\int_0^1 k(x,t)N[u_0(t)]dt \qquad ...(19)$$

$$u_{n+1}(x)=u_0(x)+\lambda\frac{x^2}{2!}\int_0^1 k(x,t)N[u_n(t)]dt\ \ n\geq 1 \qquad ...(20)$$

solution approximant $\phi_{m+1}(x)$ is given by:

$$\phi_{m+1}(x) = \sum_{n=0}^{m} u_n(x), \qquad ...(21)$$

That gives:

$$\lim_{m\to\infty} \phi_{m+1}(x) = \sum_{n=0}^{m} u_n(x) = u^*(x). \qquad ...(22)$$

Modifications of the Temimi Ansari Method for Third-order FIDEs

Consider the nonlinear third-order Fredholm integro-differential equation:

$$u'''(x) = f(x) + \lambda \int_0^1 k(x,t) N[u(t)]dt \text{ , } u(0) = c_0 \text{ , } u'(0) = c_1 \text{ , } u''(0) = c_2 \qquad ...(23)$$

Applying the three-fold integral operator $L^{-1}(.) = \int_0^x \int_0^x \int_0^x (.)dtdtdt$ to both sides of Eqn. (23) and using the given initial conditions, we obtain:

$$u(x) = c_0 + c_1 x + c_2 \frac{x^2}{2!} + \int_0^x \int_0^x \int_0^x f(t)dtdt + \lambda \frac{x^3}{3!} \int_0^1 k(x,t) N[u(t)]dt \qquad ...(24)$$

Eqn. (24) becomes

$$\sum_{n=0}^{\infty} u_n(x) = c_0 + c_1 x + c_1 \frac{x^2}{2!} + \int_0^x \int_0^x \int_0^x f(t)dtdt + \lambda \frac{x^3}{3!} \int_0^1 k(x,t) N[u_n(t)]dt \qquad ...(25)$$

Consequently, we set the following modified recursion scheme:

$$u_0(x) = f(x), \qquad ...(26)$$

Where $f(x) = c_0 + c_1 x + c_1 \frac{x^2}{2!} + \int_0^x \int_0^x \int_0^x f(t)dtdt$

$$u_1(x) = u_0(x) + \lambda \frac{x^3}{3!} \int_0^1 k(x,t) N[u_0(t)]dt \qquad ...(27)$$

$$u_{n+1}(x) = u_0(x) + \lambda \frac{x^3}{3!} \int_0^1 k(x,t) N[u_n(t)]dt \quad n \geq 1 \qquad ...(28)$$

For the determination of the solution components $u_n(x)$, $n \geq 0.$ in view of this, the solution approximant $\phi_{m+1}(x)$ is given by:

$$\phi_{m+1}(x) = \sum_{n=0}^{m} u_n(x), \qquad \text{...(29)}$$

That gives:

$$\lim_{m\to\infty} \phi_{m+1}(x) = \sum_{n=0}^{m} u_n(x) = u^*(x). \qquad \text{...(30)}$$

RESULTS AND DISCUSSION

To demonstrate the efficiency of the Modified Temimi-Ansari Method (MTAM), we apply it to solve a nonlinear Fredholm integro-differential equation as outlined in the methodology section. The following example illustrates the applicability and performance of MTAM:

Nonlinear Fredholm Integro-Differential Equation

We consider the solution of a nonlinear Fredholm integro-differential equation of the second kind to showcase the effectiveness of the proposed method.

Example 1:

As a first example, we examine a first-order nonlinear Fredholm integro-differential equation of the second kind, previously studied by Wazwaz (2011).

$$u'(x) = -\frac{47}{45} - \frac{193}{90}x + \frac{1}{12}\int_{-1}^{1}(x-t)u^2(t)dt\,, u(0) = 1 \qquad \text{...(31)}$$

With exact solution

$$u(x) = 1 - x - x^2 \qquad \text{...(32)}$$

Integrating both sides of Eqn. (31) from 0 to x and applying the initial condition, we obtain the following:

$$u(x) = 1 - \frac{47}{45}x - \frac{193}{180}x^2 + \frac{1}{12}\int_0^x\int_0^1(x-t)u^2(t)dtdt \qquad \text{...(33)}$$

Then, we define the following relations:

$u_0(x) = 1$

$$u_1(x) = 1 - \frac{47}{45}x - \frac{193}{180}x^2 + \frac{1}{12}\int_0^x\int_0^1 (x-t)u_0^{\,2}(t)dtdt = 1 - \frac{47}{45}x - \frac{163}{180}x^2$$

$$u_2(x) = 1 - \frac{47}{45}x - \frac{193}{180}x^2 + \frac{1}{12}\int_0^x\int_0^1 (x-t)u_1^{\,2}(t)dtdt = 1 - \frac{120461}{121500}x - \frac{2677573}{2916000}x^2$$

$$u_3(x) = 1 - \frac{47}{45}x - \frac{193}{180}x^2 + \frac{1}{12}\int_0^x\int_0^1 (x-t)u_2^{\,2}(t)dtdt =$$

$$1 - \frac{5287708661153}{5314410000000}x - \frac{235925881969511}{255091680000000}x^2$$

$$u_4(x) = 1 - \frac{47}{45}x - \frac{193}{180}x^2 + \frac{1}{12}\int_0^x\int_0^1 (x-t)u_3^{\,2}(t)dtdt =$$

$$1 - \frac{20238124329978094534176106183}{20334926626632000000000000000}x - \frac{1805365158103105182191515090319}{1952152956156672000000000000000}x^2$$

$$u_5(x) = 1 - \frac{47}{45}x - \frac{193}{180}x^2 + \frac{1}{12}\int_0^x\int_0^1 (x-t)u_4^{\,2}(t)dtdt =$$

$$1 - \frac{5926085791303821938749010698425126342695488375138888979342377}{595453306911130211229595330560000000000000000000000000000000}x - \frac{105726793492529433167691706275102052027359392136089334131527199}{114327034926937000556082303467520000000000000000000000000000000}x^2$$

And so on. The five-term approximate solution is

$$u(x) =$$

$$1 - \frac{5926085791303821938749010698425126342695488375138888979342377}{595453306911130211229595330560000000000000000000000000000000}x - \frac{105726793492529433167691706275102052027359392136089334131527199}{114327034926937000556082303467520000000000000000000000000000000}x^2 \quad ...(34)$$

Table 1: Numerical solution of MTAM, VIM, ADM and Exact solution for Eqn. (31)

X	EXACT	MTAM (n=5)	VIM (n=7)	ADM (n=6)
0.0	1.00	1.00000000	1.0000000	1.0000000
0.1	0.89	0.89122999	0.8916728	0.8912538
0.2	0.76	0.76396448	0.7649809	0.7640102
0.3	0.61	0.61820347	0.6199243	0.6182693
0.4	0.44	0.45394695	0.4565030	0.4540310
0.5	0.25	0.27119494	0.2747170	0.2712954
0.6	0.04	0.06994742	0.0745663	0.0700625
0.7	-0.19	-0.1497956	-0.1439490	-0.1496680
0.8	-0.44	-0.38803412	-0.3808290	-0.3878950
0.9	-0.71	-0.64476815	-0.6360740	-0.6446200
1.0	-1.00	-0.91999767	-0.9096830	-0.9198430

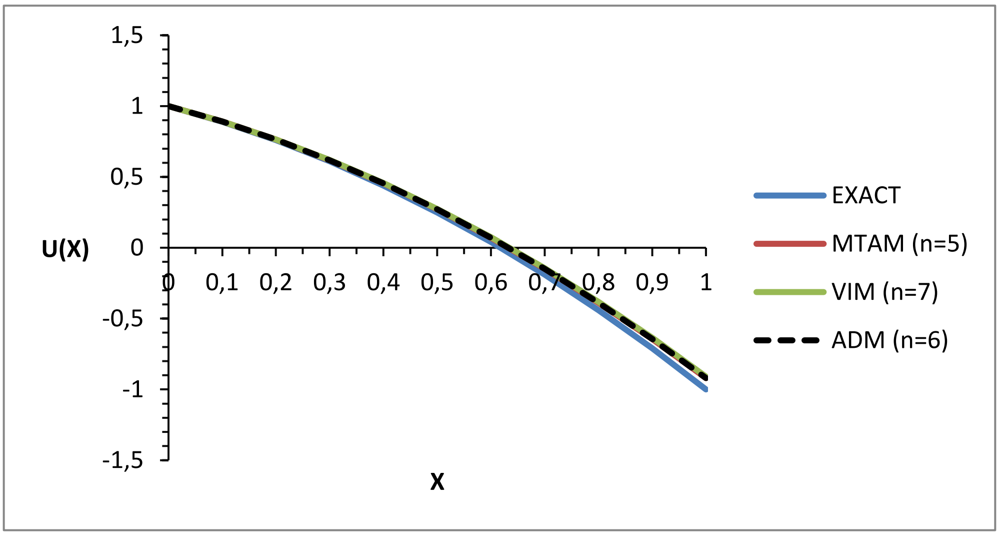

Figure 1: **Plots of approximate solution of MTAM, VIM, ADM and exact solution for Eqn. (31)**

Table 2: Approximate solution of Eqn. (31) for Five-term, Four-term, Three-term and two terms

X	EXACT	MTAM (n=5)	MTAM (n=4)	MTAM (n=3)	MTAM (n=2)
0	1	1	1	1	1
0.1	0.89	0.89122999	0.89122797	0.89125376	0.891672795
0.2	0.76	0.76396448	0.76395979	0.76401018	0.764980892
0.3	0.61	0.61820347	0.61819547	0.61826927	0.61992429
0.4	0.44	0.45394695	0.453935	0.45403101	0.45650299
0.5	0.25	0.27119494	0.27117839	0.27129541	0.274716992
0.6	0.04	0.06994742	0.06992564	0.07006247	0.074566296
0.7	-0.19	-0.1497956	-0.14982326	-0.14966781	-0.1439491
0.8	-0.44	-0.3880341	-0.38806831	-0.38789543	-0.38082919
0.9	-0.71	-0.6447681	-0.6448095	-0.64462039	-0.63607398
1	-1	-0.9199977	-0.92004683	-0.91984269	-0.90968347

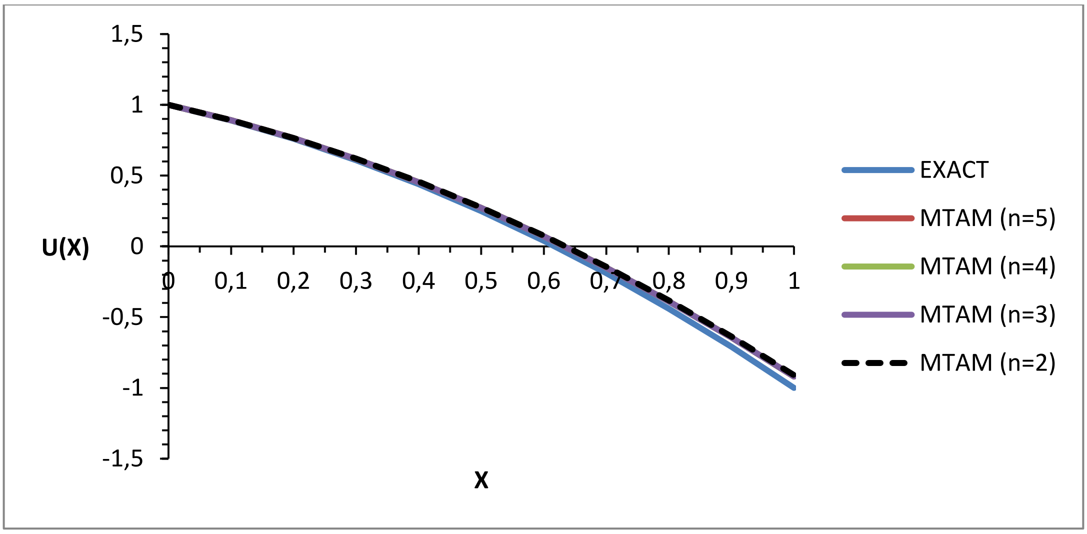

***Figure 2*: Plots of Approximate solution of Eqn. (31) for Five-term, Four-term, Three-term and two terms by the MTAM**

Figure 1 shows the numerical solution for the exact solution, MTAM, VIM and ADM obtains by using MAPLE18 software. The graph of the MTAM is in better agreement with the exact solution than the VIM and ADM. In Figure 2, we plot the approximate solution of

Eqn. (31) for Five-term, Four-term, Three-term and two terms by the MTAM. This clearly shows that the approximate solution got better by adding more terms.

Example 2:

The second example is a nonlinear second-order FIE of the second kind (Rani & Mishra, 2019) and (Wazwaz, 2011)

$$u''(x) = \sinh(x) + x - \int_0^1 x(\cosh^2(t) - u^2(t))dt\,,\quad u(0)=0,\ \ u'(0)=1 \qquad ...(35)$$

Having the exact solution as $u(x) = \sinh(x)$

The equation can be written in the operator form as

$$u(x) = \sinh(x) + \frac{x^3}{6} - \frac{x^3}{6}\int_0^1 \cosh^2(t)dt + \frac{x^3}{6}\int_0^1 u^2(t)dt \qquad ...(36)$$

Then, we define the following relations:

$$u_0(x) = 0$$

$$u_1(x) = \sinh(x) + \frac{x^3}{6} - \frac{x^3}{6}\int_0^1 \cosh^2(t)dt + \frac{x^3}{6}\int_0^1 u_0^{\ 2}(t)dt = \sinh(x) - 0.06780x^3$$

$$u_2(x) = \sinh(x) + \frac{x^3}{6} - \frac{x^3}{6}\int_0^1 \cosh^2(t)dt + \frac{x^3}{6}\int_0^1 u_1^{\ 2}(t)dt = \sinh(x) - 0.00499x^3$$

$$u_3(x) = \sinh(x) + \frac{x^3}{6} - \frac{x^3}{6}\int_0^1 \cosh^2(t)dt + \frac{x^3}{6}\int_0^1 u_2^{\ 2}(t)dt = \sinh(x) - 0.00039x^3$$

$$u_4(x) = \sinh(x) + \frac{x^3}{6} - \frac{x^3}{6}\int_0^1 \cosh^2(t)dt + \frac{x^3}{6}\int_0^1 u_3^{\ 2}(t)dt = \sinh(x) - 0.00005x^3$$

$$u_5(x) = \sinh(x) + \frac{x^3}{6} - \frac{x^3}{6}\int_0^1 \cosh^2(t)dt + \frac{x^3}{6}\int_0^1 u_4^{\ 2}(t)dt = \sinh(x) - 0.00002x^3$$

And so on. The five-term approximate solution is

$$u(x) = \sinh(x) - 0.00002x^3 \qquad ...(37)$$

Table 3: Numerical solution of MTAM, ADM, DTM and Exact solution for Eqn. (35)

X	EXACT	MTAM(n=5)	ADM (n=8)	DTM(n=4)
0	0	0	0	0
0.1	0.1001668	0.10016673	0.10016636	0.1001618
0.2	0.201336	0.201335843	0.201332883	0.2012961
0.3	0.3045203	0.304519753	0.304509763	0.3043856
0.4	0.4107523	0.410751046	0.410727366	0.410433
0.5	0.5210953	0.521092805	0.521046555	0.5204716
0.6	0.6366536	0.636649262	0.636569342	0.6355757
0.7	0.7585837	0.758576842	0.758449932	0.7568721
0.8	0.888106	0.888095742	0.887906302	0.8855511
0.9	1.0265167	1.026502146	1.026232416	1.022879
1	1.1752012	1.175181194	1.174811194	1.1702112

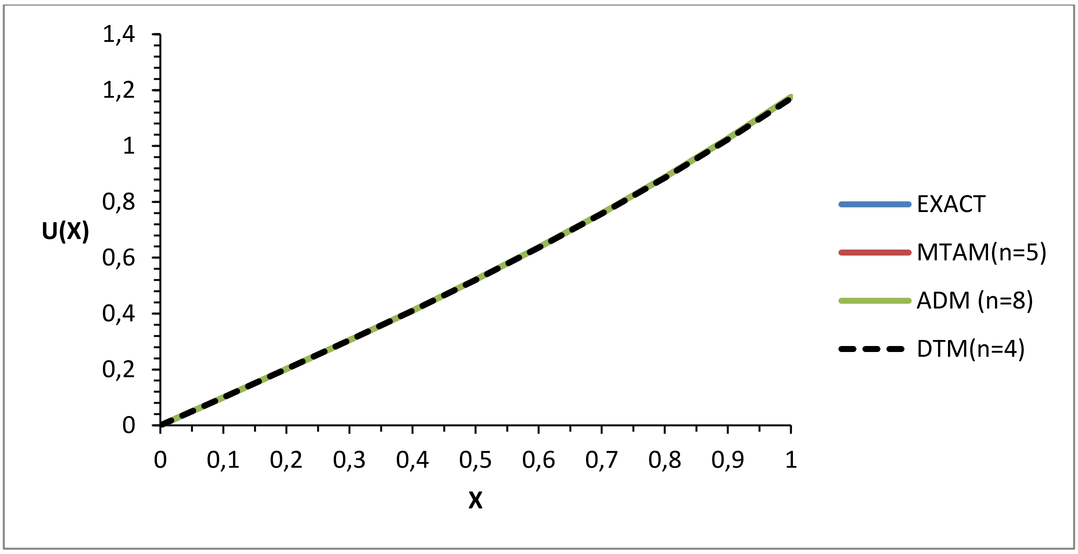

***Figure 3*: Plots of approximate solution of MTAM, ADM, DTM and exact solution for Eqn. (35)**

Table 4: Approximate solution of the MTAM for Five-term, Four-term, Three-term and Two-term for Eqn. (35)

X	EXACT	MTAM(n=5)	MTAM(n=4)	MTAM(n=3)	MTAM(n=2)
0	0	0	0	0	0
0.1	0.100167	0.10016673	0.1001667	0.10016636	0.10016176
0.2	0.201336	0.20133584	0.2013356	0.20133288	0.20129608
0.3	0.30452	0.30451975	0.30451894	0.30450976	0.30438556
0.4	0.410752	0.41075105	0.41074913	0.41072737	0.41043297
0.5	0.521095	0.52109281	0.52108906	0.52104656	0.52047156
0.6	0.636654	0.63664926	0.63664278	0.63656934	0.63557574
0.7	0.758584	0.75857684	0.75856655	0.75844993	0.75687213
0.8	0.888106	0.88809574	0.88808038	0.8879063	0.8855511
0.9	1.026517	1.02650215	1.02648028	1.02623242	1.02287902
1	1.175201	1.17518119	1.17515119	1.17481119	1.17021119

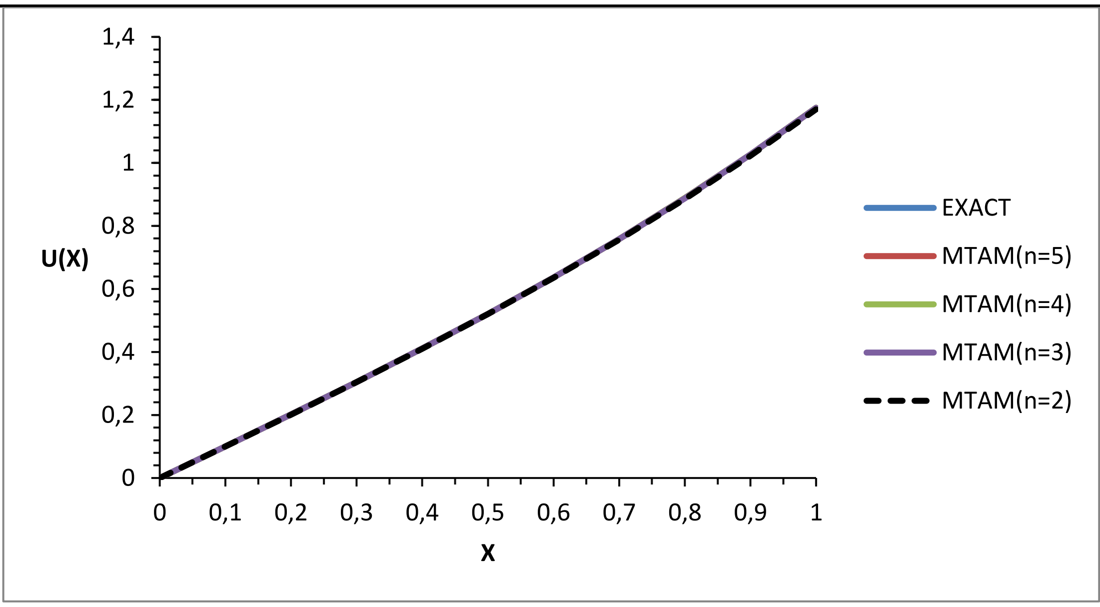

***Figure 4*: Plots of approximate solution of the MTAM for Five-term, Four-term, Three-term and Two-term of Eqn. (35)**

Table 5: Absolute errors for Eqn. (35) for the MTAM, ADM and DTM

X	EXACT	MTAM(n=5)	ADM(n=8)	DTM(n=4)	MTAM Error	ADM Error	DTM Error
0	0	0	0	0	0	0	0
0.1	0.1001668	0.10016673	0.10016636	0.1001618	2E-08	3.9E-07	4.99E-06
0.2	0.201336	0.201335843	0.201332883	0.2012961	1.6E-07	3.12E-06	3.992E-05
0.3	0.3045203	0.304519753	0.304509763	0.3043856	5.4E-07	1.053E-05	0.0001347
0.4	0.4107523	0.410751046	0.410727366	0.410433	1.28E-06	2.496E-05	0.0003194
0.5	0.5210953	0.521092805	0.521046555	0.5204716	2.5E-06	4.875E-05	0.0006238
0.6	0.6366536	0.636649262	0.636569342	0.6355757	4.32E-06	8.424E-05	0.0010778
0.7	0.7585837	0.758576842	0.758449932	0.7568721	6.86E-06	0.0001338	0.0017116
0.8	0.888106	0.888095742	0.887906302	0.8855511	1.024E-05	0.0001997	0.0025549
0.9	1.0265167	1.026502146	1.026232416	1.022879	1.458E-05	0.0002843	0.0036377
1	1.1752012	1.175181194	1.174811194	1.1702112	2E-05	0.00039	0.00499

Figure 3 shows the numerical solution for the exact solution, MTAM, ADM and DTM obtains by using MAPLE18 software. The graph of the MTAM is seen to converge faster than that of ADM and DTM. In Figure 4, we plot the approximate solution of Eqn. (35) for Five-term, Four-term, Three-term and two terms by the MTAM. This clearly shows that the approximate solution got better by adding more terms. In Table 5, the absolute error for MTAM, ADM and DTM shows that the numerical solution for both methods is close to the exact solution. But, by comparing the absolute error by the methods, the minimum absolute error goes to MTAM. This shows that MTAM is more accurate than the ADM and DTM.

CONCLUSION

We have presented and analyzed the MTAM for solving nonlinear Fredholm integro-differential equation of second kind. We have applied the MTAM to solve some physical problems and obtained closed form as well as exact solutions for these problems. It was observed that once the initial approximation is identified, with the proper use of the MTAM it is possible to obtain analytical solution to a class of linear and nonlinear Fredholm integro-differential equations. In our quest to find and discover if this method can be applicable for solving the second order FIDEs, with few examples used we have successfully demonstrated the efficiency, accuracy and convergence of this method. We have also confirmed that, the use of MTAM reduces the volume of calculations by not requiring the use of computer program or polynomials for nonlinear terms.

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