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Mathematical Modeling of Security Forces – Insurgent Dynamics in Nigeria

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Abstract

In this study, a mathematical model of the dynamics of the interaction between Nigerian security forces and armed groups was developed. This model is based on demographic principles. When developing the model, the dynamics were conceptualized and structured along with the dynamics of predators and prey. The model developed was an analysis based on the Routh-Hurtwiz standard. The equilibrium points of the model were determined and their stability analysis was performed. The equilibrium factor is domestically asymptotically stable. In addition, we conducted numerical experiments to simulate the solution of the model. This study suggests that security agencies should be proactive in their response and improve their intelligence, peace building and weapons skills in combat conflicts in order to motivate security forces. Strengthen security forces and rehabilitation centers and improve rehabilitation programs for society as a whole.

Keywords: Boko Haram, Insurgent, Rehabilitation and stability

Introduction

The worlds are struggling with uproar of insurgency. The phrase insurgency usually appeals to the spirit of evil, terror and rebellionin competition to an incumbent government and to undermine the strive of constituted authority. The Giant of Africa "Nigeria" is witnessing havoc unleashing via way of means ofpositive organization referred to as Boko Haram. And various crimes are been reported almost every day basis as Taraba State was not excluded due to the infectious transmission of crime rate recorded by United nations office on drugs and crimes (UNODC) Akpienbi *et al,* (2021).The Boko Haram Islamic fundamentalist wing label has hyperlinks with any other terrorist organization Okoroafor&Ukpabi, (2015). Their poor effect is felt in specific elements of the sector inclusive of Africa Okemi, (2013). Terrorism is a danger and crime in opposition to humanity, harmless civilians and combatant individual, organization or country agents Chanchal, (2012). Possession of chemicals, radioactive materials, or explosives, hydro technical systems is a high-quality chance to the society Chanchal, (2012).

The Boko Haram insurgency in Nigeria has had a variety of socio-political and monetary outcomes in Nigeria Durotoye, (2015). A mathematical version became evolved to look at the dynamics of protection forces and crook sports with the assist of voluntary reinforcement. Oduro, (2015). Terrorist groups alternate through the years due to procedures inclusive of recruitment and education in addition to counter-terrorism (CT) measures Gutfraind (2010). To combat in opposition to them the network takes specific safety features Juan, Miguel, and Mario, (2000). First decide the geographical profile in which the crook can also additionally disguise and predictions comprise the places in which the assassin is maximum possibly to stay with its approximate variety and the viable time of the subsequent crime Feroz Shah Syed, and Zhenhong (2013). It is thought that the Security forces take`s into custody all insurgents` factors with inside the society. Thus, while modeling insurgents, there's excessive diploma of compromise because of assumptions, additionally for a very good purpose that human conduct is inherently nonlinear, so we count on that they will great be defined via way of means of a nonlinear system, every now and then via way of means of Lotka-Volterra version modification.

Model Formulation

The populace is compartmentalized into protection forces (security forces), insurgents and rehabilitated compartments. Let X, S and R denoted the variety of insurgents, protection forces and rehabilitated at time t.In the presence of insurgents, protection forces could be deployed with the motive of killing and shooting the insurgents for rehabilitation with the subsequent dynamics; The rebel`s populace will increase throughσX because of recruitment of people into the rebel and reduce through;αXSbecause of recruitment of people into the rebel and reduce through; αXS because of the interplay with the safety forces and reduces through$\tau u_1 X$ because of touch fee of insurgents with volunteer guards/civilian JTF, ωXS the captured and rehabilitated rebel`s and $\theta X$ herbal dying fee. The rehabilitated insurgent's boom through $\omega XS$ because of rehabilitation of captured and reduces through herbal dying fee. The protection forces improved through $\gamma XS$ because of interplay with the rebel`s might result in call for extra protection forces, and$\tau u_1 X$ because of recruitment of volunteer guard/civilian JTF and reduces through βS because of retirement or withdrawer of protection forces in absences of rebel`s and herbal dying fee $\theta S$. The following assumptions are used for formula of the version. In the absence of any protection pressure, rebel grows exponentially; the interplay among the safety forces and insurgents is random, the insurgents` sports make a contribution to the safety pressure increase fee, if no protection forces are gift with inside the network insurgents perform with inside the network at a fee proportional to their populace, the increase fee of insurgents decreases through a thing proportional to the variety of come upon among the safety forces and the insurgents, with inside the absence of insurgents the safety forces populace has a tendency to decline, seize rebels could be rehabilitated. Using the above assumptions and the dynamical glide diagram description, the subsequent machine of non-linear differential equations (1) – (3) to control the version became derived below.

$$\frac{dX}{dt} = \sigma X - \alpha XS - \tau \mu_1 X - \omega XS - \theta X \quad (1)$$

$$\frac{dS}{dt} = -\beta S + \gamma XS + \tau \mu_1 X - \theta S \quad (2)$$

$$\frac{dR}{dt} = \omega XS - \theta R \quad (3)$$

Let $\frac{dX}{dt}$, $\frac{dS}{dt}$ and $\frac{dR}{dt}$ should be $\frac{dx}{dt}$, $\frac{ds}{dt}$ and $\frac{dr}{dt}$ for easy simplification

Where $x(0) > 0,\ s(0) > 0$and $r(0) > 0$

Analytical Results

We investigated the developed mathematical model of security forces, the dynamics of the rebels. The equilibrium state of the system was obtained by setting the model equations to zero and solving the resulting algebraic equations at the same time.

1. Insurgent free equilibrium point E_1

From equation (1) we let $x = 0$ and substitute the value of x into model equations (2) and (3) we have $s = 0$ and $r = 0$. Hence $E_1(0,0,0)$ the equilibrium point.

2. Coexistence equilibrium point E_2

From equation (4.1) let make s subject

$$\Rightarrow s = \frac{\sigma - \theta - \tau\mu_1}{\alpha + \omega}$$

Substitute s into equation (2) and we have

$$x = \frac{(\sigma - \theta - \tau\mu_1)(\theta + \beta)}{\gamma(\sigma - \theta - \tau\mu_1) + \tau\mu_1(\alpha + \omega)}$$

Similarly, let substitute value of x and s in equation (3) and we have

$$r = \omega(\frac{(\theta+\beta)}{(\alpha+\omega)})\frac{(\theta-\sigma+\tau\mu_1)^2}{(\sigma\gamma-\theta\gamma+\alpha\tau\mu_1-\tau\gamma\mu_1+\tau\omega\mu_1)}$$

Therefore, $E_2 = (\frac{(\sigma-\theta-\tau\mu_1)(\theta+\beta)}{\gamma(\sigma-\theta-\tau\mu_1)+\tau\mu_1(\alpha+\omega)}, \frac{\sigma-\theta-\tau\mu^1}{\alpha+\omega}, \omega(\frac{(\theta+\beta)}{(\alpha+\omega)})\frac{(\theta-\sigma+\tau\mu_1)^2}{(\sigma\gamma-\theta\gamma+\alpha\tau\mu_1-\tau\gamma\mu_1+\tau\omega\mu_1)})$

3. Local Stability of the Model

The Routh-Hurbitz condition was used to determine the local approach stability of equilibrium. The Routh Hurwitz criterion shows the necessary and sufficient condition that the equation $p(\lambda) = \lambda^n + a_1\lambda^{n-1} + \cdots + a_n = 0,$ (using the real factor) has only zeros with negative real parts. .. All the values of the determinant of the matrix are positive.

Now, if we get the partial derivatives of model equations (1) to (3), we get a matrix of the form:

$$J_E = \begin{bmatrix} \sigma - (\alpha+\omega)s - \tau\mu_1 - \theta & -(\alpha+\omega)x & 0 \\ \gamma s + \tau\mu_1 & -\beta + \gamma x - \theta & 0 \\ \omega s & \omega x & -\theta \end{bmatrix} \quad (4)$$

4 Stability analysis of free insurgent's equilibrium point of the model

We substitute the equilibrium point E_1 into the modified model matrix (4), we have

$$J_{E_1} = \begin{bmatrix} \sigma - \tau\mu_1 - \theta & 0 & 0 \\ \tau\mu_1 & -\beta - \theta & 0 \\ 0 & 0 & -\theta \end{bmatrix}$$

Then,

$$det\left|J_{E_1} - \lambda I\right| = \begin{vmatrix} \sigma - \tau\mu_1 - \theta - \lambda & 0 & 0 \\ \tau\mu_1 & -\beta - \theta - \lambda & 0 \\ 0 & 0 & -\theta - \lambda \end{vmatrix} = 0$$

And we obtained a characteristic polynomial as

$$\lambda^3 + (3\theta - \sigma + \beta + \tau\mu_1)\lambda^2 + (3\theta^2 - 2\theta\sigma + 2\theta\beta - \sigma\beta + 2\theta\tau\mu_1 + \beta\tau\mu_1)\lambda + (\theta^3 - \theta^2\sigma + \theta^2\beta + \theta^2\tau\mu_1 - \theta\sigma\beta + \theta\beta\tau\mu_1) \quad (5)$$

Equation (4.8) as $A_3\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 = 0$

Where A_3=1, A_2=$(3\theta - \sigma + \beta + \tau\mu_1)$, A_1=$(3\theta^2 - 2\theta\sigma + 2\theta\beta - \sigma\beta + 2\theta\tau\mu_1 + \beta\tau\mu_1)$, A_0=$(\theta^3 - \theta^2\sigma + \theta^2\beta + \theta^2\tau\mu_1 - \theta\sigma\beta + \theta\beta\tau\mu_1)$

Then apply the Routh-Hurwitz condition. This shows that all zeros of the polynomial has a negative real part only if the coefficient D_i is positive and is the determinant of the matrix.

$A_i > 0$ for $i = 0, 1, 2, 3, 4$.

$A_1 > 0, A_2 = \begin{vmatrix} D_3 & D_1 \\ 1 & D_2 \end{vmatrix} = D_2D_3 - D_1 > 0$, if and only if $D_2D_3 > D_1$

Then, $A_2 > 0, D_3 = \begin{vmatrix} D_3 & D_1 & 0 \\ 1 & D_2 & D_0 \\ 0 & D_3 & D_2 \end{vmatrix} = D_1D_2D_3 - D_0{D^2}_3 - {D^2}_1$

$D_1D_2D_3 - (D_0{D^2}_3 + {D^2}_1) > 0$, if and only if $D_1D_2D_3 > D_0{D^2}_3 + {D^2}_1$

Thus, $A_3 > 0$

Therefore, all the roots of the polynomial (5) have negative real parts, implying that $\lambda_i < 0$, for $i = 1, 2, 3, 4.$ we conclude that the equilibrium point is locally asymptotically stable.

5. Stability analysis of coexistence equilibrium point

We substitute the equilibrium point E_2 into the modified model Jacobian matrix equation (4)

$$J_{E_2} = \begin{bmatrix} \sigma - (\alpha+\omega)s - \tau\mu_1 - \theta & -(\alpha+\omega)x & 0 \\ \gamma s + \tau\mu_1 & -\beta + \gamma x - \theta & 0 \\ \omega s & \omega x & -\theta \end{bmatrix}$$

Then,

$$det\left|J_{E_2} - \lambda I\right| = \begin{vmatrix} \sigma - (\alpha+\omega)s - \tau\mu^1 - \theta - \lambda & -(\alpha+\omega)x & 0 \\ \gamma s + \tau\mu_1 & -\beta + \gamma x - \theta - \lambda & 0 \\ \omega s & \omega x & -\theta - \lambda \end{vmatrix} = 0$$

Thus, we obtained characteristic polynomial as $\lambda^3 + (2\theta + \beta\frac{(\theta+\beta)}{\gamma(\theta-\sigma+\tau\mu_1)-\tau\mu_1(\alpha+\omega)}(\theta - \sigma + \tau\mu_1))\lambda^2 + (\theta(\theta + \beta\frac{(\theta+\beta)}{(\gamma(\theta-\sigma+\tau\mu_1)-\tau\mu_1(\alpha+\omega)}(\theta - \sigma + \tau\mu_1)) + (\theta + \beta)(\frac{(\alpha+\omega)}{(\gamma(\theta-\sigma+\tau\mu^1)-\tau\mu^1(\alpha+\omega))})(\tau\mu^1 - \frac{(\gamma}{(\alpha+\omega)})(\theta - \sigma + \tau\mu_1))(\theta - \sigma + \tau\mu_1))\,\lambda + \theta(\theta + \beta)(\frac{(\alpha+\omega)}{(\gamma(\theta-\sigma+\tau\mu_1)-\tau\mu_1(\alpha+\omega)})(\tau\mu_1 - (\frac{\gamma}{\alpha+\omega})(\theta - \sigma + \tau\mu_1))\,(\theta - \sigma + \tau\mu_1)$

(6)

And equation (6) can be written as $A_4\lambda^4 + A_3\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 = 0$

Where$A_3 = 1, A_2 = \left(2\theta + \beta\frac{(\theta+\beta)}{\gamma(\theta-\sigma+\tau\mu_1)-\tau\mu_1(\alpha+\omega)}(\theta - \sigma + \tau\mu_1)\right), A_1 = (\theta(\theta + \beta\frac{(\theta+\beta)}{(\gamma(\theta-\sigma+\tau\mu_1)-\tau\mu_1(\alpha+\omega)}(\theta - \sigma + \tau\mu_1)) + (\theta + \beta)(\frac{(\alpha+\omega)}{(\gamma(\theta-\sigma+\tau\mu_1)-\tau\mu_1(\alpha+\omega))})(\tau\mu_1 - \frac{(\gamma}{(\alpha+\omega)})(\theta - \sigma + \tau\mu_1))(\theta - \sigma + \tau\mu_1)), A_0 = \theta(\theta + \beta)(\frac{(\alpha+\omega)}{(\gamma(\theta-\sigma+\tau\mu_1)-\tau\mu_1(\alpha+\omega)})(\tau\mu_1 - (\frac{\gamma}{\alpha+\omega})(\theta - \sigma + \tau\mu_1))\,(\theta - \sigma + \tau\mu_1)$

We then apply a Routh-Hurwitz condition which states that all roots of the polynomial have negative real part if and only if the coefficients D_i are positive and the determinant of the matrices $A_i > 0$ for $i = 0, 1, 2, 3, 4$.

$A_1 > 0, A_2 = \begin{vmatrix} D_3 & D_1 \\ 1 & D_2 \end{vmatrix} = D_2D_3 - D_1 > 0$, if and only if $D_2D_3 > D_1$

$$A_2 > 0, A_3 = \begin{vmatrix} D_3 & D_1 & 0 \\ 1 & D_2 & D_0 \\ 0 & D_3 & D_2 \end{vmatrix} = D_1 D_2 D_3 - D_0 D^2{}_3 - D^2{}_1$$

Which indicate that $D_1 D_2 D_3 - (D_0 D^2{}_3 + D^2{}_1) > 0$, if and only if$D_1 D_2 D_3 > D_0 D^2{}_3 + D^2{}_1$

$A_3 > 0$

Therefore, all the roots of the polynomial (6) have negative real parts, implying that $\lambda_1 < 0$, $\lambda_2 < 0$, $\lambda_3 < 0$, $\lambda_4 < 0$. Hence, since all the values of $\lambda_i < 0$, for $i = 1, 2, 3, 4$. we conclude that the insurgents free equilibrium point is locally asymptotically stable.

6. Numerical experiment

Numerical simulations were performed using the parameter values in Table 1 and the initial population of (x, y, r). The code was implemented in the MATLAB 2012b software and generated a diagram of population trends. The results of the numerical simulation are displayed after changing the initial population of the model to get the graph. Some parameters are taken from Elettreby (2007) and Nijamuddin (2007) and are listed in Table 2.

Table 1: parameter values and initial variable

Variable/parameter	**Value**	**Reference**
x	0.5, 0.2, 0.3	Assumed
s	0.3, 0.3, 0.5,	Assumed
r	0.2, 0.5, 0.2	Assumed
γ	0, 3.0.1, 0.06	Eletterby*et.al*, (2007)
ƭ	0.2, 0.32	Nijamuddin*et.al*, (2007)
θ	0.02, 0.07	Eletterby*et.al*, (2007)
α	0.4, 0.12	Nijamuddin*et.al*, (2007)
μ_1	0.3, 0.03	Nijamuddin*et.al*, (2007)
ω	0.3, 0.2, 0.23	Nijamuddin*et.al*, (2007)
σ	0.2, 0.02, 0.04	Nijamuddin*et.al*, (2007)
β	0.3 2	Assumed

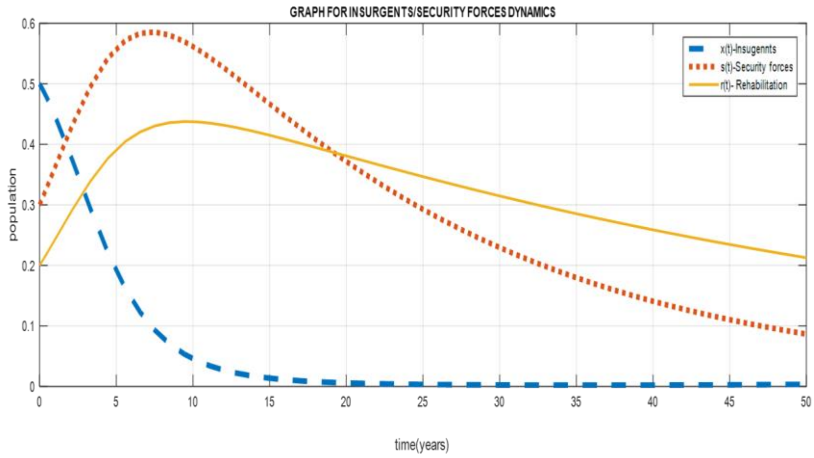

Figure a: The model behavior

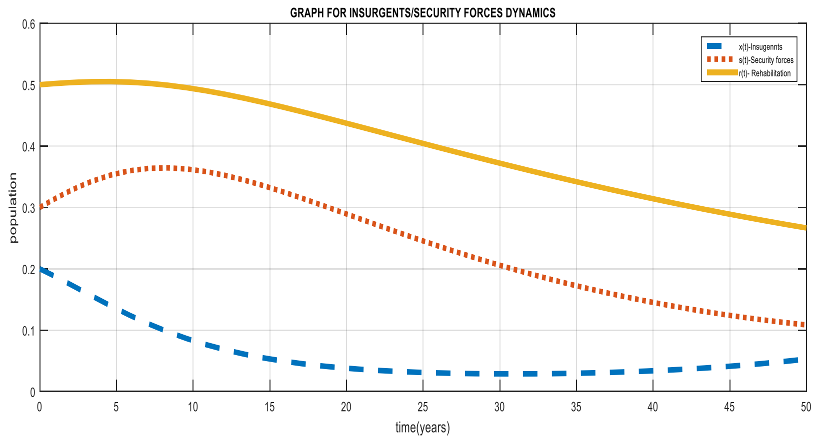

Figure b: The model behavior

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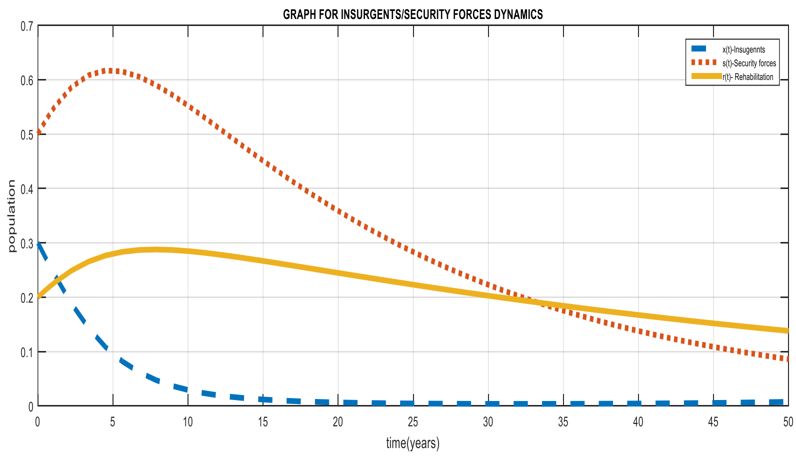

Figure c: The model behavior

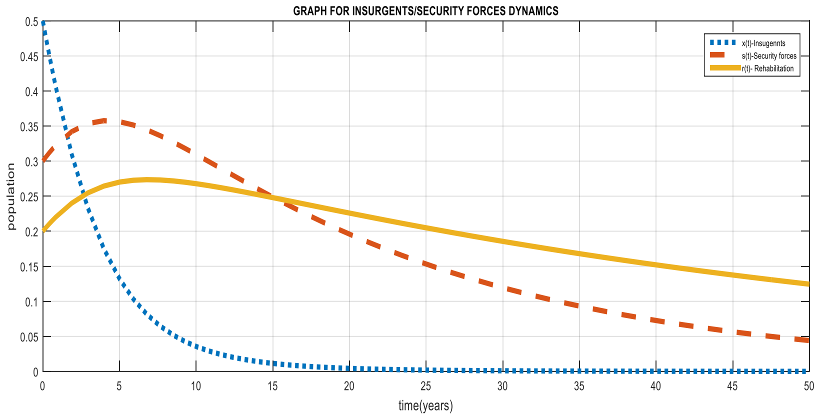

Figure d: The model behavior

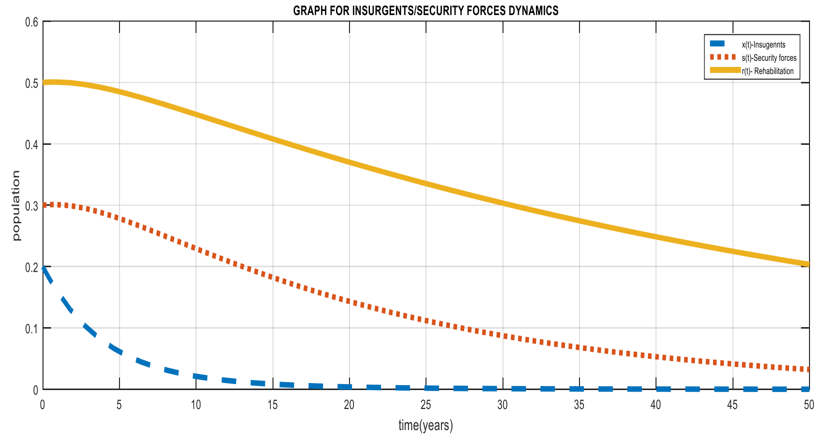

Figure e : The model behavior

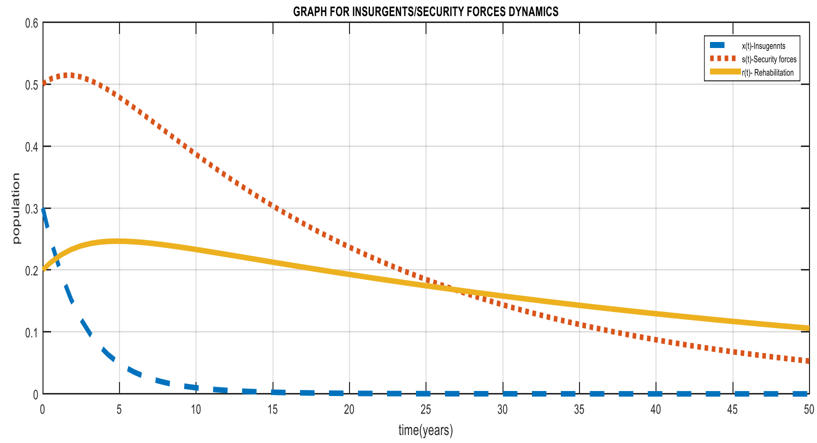

Figure f: The model behavior

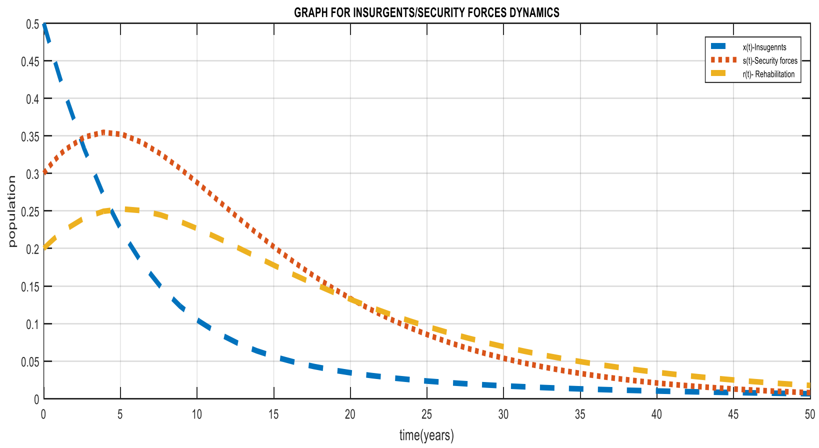

Figure g: The model behavior

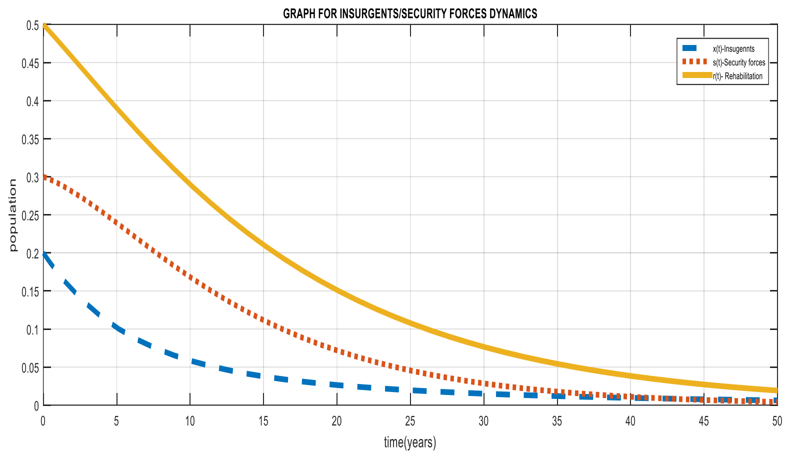

Figure h: The model behavior

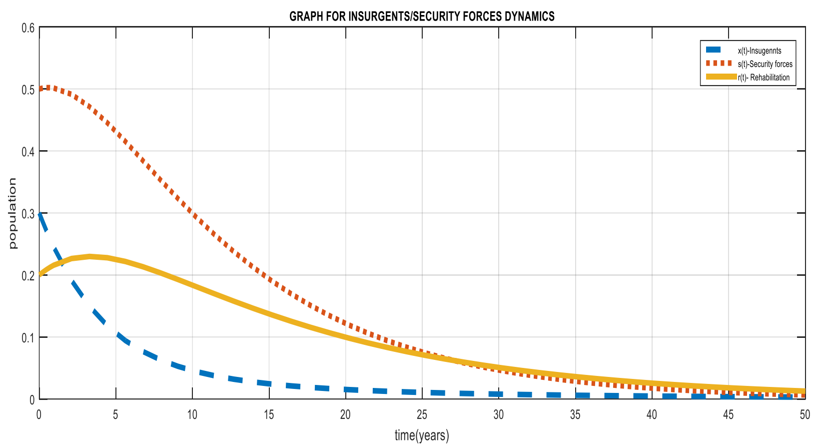

Figure i: The model behavior

Discussion of Results

This section describes the results of analytical studies and numerical experiments.

Analytic Result

The mathematical model developed was a system of three-dimensional ODEs. Non-riot and coexistence equilibrium are for the systems of equations (1) to (3). The Routh-Hurbitz conditions for linearization and stability were used to determine the local stability of the model's equilibrium points. We saw that all the eigen-values are negative, that is, $\lambda_i < 0$for $i = 1, 2, 3, 4$. That is, $D_1 > 0,\ D_2 > 0,$ and $D_3 > 0$.this means that the equilibrium point is locally and asymptotically stable.

The results of the numerical simulation are shown in Figure a-h. These curves were created using the MATLAB 2012b software. To analyze the sensitivity of each parameter in the model, the parameter values changed in each case of the simulation, and so did the initial population.

These representations envisioned a fictitious population. Figures (a), (b), and (c) all show that as the security force population grows, the rebel population loses stability, and when it crosses a rehab program, it declines towards stability. It will increase or decrease according to changes in the initial population. The graph in (d) shows that the number of security forces increases as the armed population increases and decreases as the armed population decreases. Similarly, rehabilitation programs increase or decrease the rebel population in both graphs (e) and (f). Security forces and rehabilitation as the armed forces have a large population and are declining in the same way that armed groups are declining towards stability.

Conclusion

In this study, a modified version of Oduro, *et al.* (2015)'s prey-predator model of security forces and criminal dynamics was proposed. The effects of different security forces intervention strategies and rehabilitation methods on the dynamics of the system were investigated. The model's equilibrium points were determined, and the model's local stability and numerical simulations were carried out.

Recommendations

These results encourage federal, state, and local governments to advocate rehabilitation programs and campaigns, motivate security agencies and staff responsible for the programs, and develop better intervention strategy packages.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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