Vector Autoregressive Modeling of Some Nigeria Inflation Factors

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Abstract

This study investigates the modeling of some macroeconomic variables in Nigerian using multivariate monthly time series data from January 2010 to December 2019. The study examined two inflation factors which include money supply (MS) and exchange rate (ER) which are of great importance in determining inflationary effect in any economy. If MS is greater than ER then it can be said that the economy is experiencing rise in inflation and vice versa. Based on the analysis of implementing a vector autoregressive model to the data at stationarity using the Augmented Dickey–Fuller (ADF) test, Inflation rate (IR) was found to be significant only at lag 2 of IR. However, MS was found to be significant at lag 1 of ER and ER was significant at lag 1 and lag 2 of ER. The impulse response function plots clearly showed an unstable IR on MS and ER but at the later end of the periods, Nigeria IR tends towards a positive stability on MS and ER, respectively.

Keywords: Vector autoregressive, Macroeconomic variables, Information criteria, Impulse response function, Inflationary effect

INTRODUCTION

Multivariate time series are series of equations that involve multiple entering equations variable of simultaneous equations models. Sims (1980) advocate the used of vector autoregressive (VAR) models as alternative to multivariate simultaneous equations models. Traditionally VAR models are designed for stationarity of variables without time trends were trend behavior can be captured by including deterministic rise to power terms. Studies on inflationary effect on some macroeconomic variables have been studied by many authors. Amongst these studies include Jayaraman *e.t al.* (2013) modeling of inflation and growth in Fiji by looking through the threshold of inflation rate. They found out that the threshold level of inflation for Fiji, based on the past trends in growth and inflation is 3.6 percent. Therefore they conclude by saying as long as the inflation level is below this threshold level, the effect on growth would be positive and higher levels would adversely affect growth.

Audu *et al.* (2015) investigated the impact of crude oil shocks on exchange rate (ER), external reserves (ERV), gross domestic product (GDP), inflation rate (IR), international Trade (IT), and money supply (MS) in Nigeria using a quarterly data from the period of 2000 to 2014. From their empirical analysis, generalized autoregressive conditional heteroscedastic (GARCH) model was employed on the variable with presence of heteroscedastic effect. The VAR model was mentioned in their methodology but without output results. Furthermore, their study impulse response function plots showed the upper and lower boundary using positive and negative two standard errors of the macroeconomic variables. However, the plot showed that ER, GDP, and MS increased in the first four quarters with the exception of IR and IT to negative crude oil prices. While in many cases, this increase has quickly shifted from decrease to stabilized phase over the successive quarters. Another study on IR effect on GDP is that of Farouk *et al.* (2021a) where they studied inflationary effect on Nigeria growth from 1986 to 2018 using autoregressive distributed lag (ARDL) model and found IR effect to be negative on Nigeria growth both in the short and long run. Adubisi *et al.* (2018) studied Nigeria IR using a predictive autoregressive integrated moving average (PARIMA) model for forecasting the rate of Nigeria inflation from January to December of 2017 and their selected PARIMA model of PARIMA (1, 2, 1) was found to be a good fitted model. Recently, David *et al.* (2024) used ARDL model to model the effect of IR, MS, and ER on Nigeria stock market prices (NSMP). Their findings revealed that IR has no impact of NSMP both in the short and

long runs. However, MS was found to have a significant positive effect, both in the long run and short run, while ER effect on NSMP is not significant in the long run, but in the short run, it was significant with a negative effect on NSMP in the second and third lags. They also, studied a combined model and it was found that at lag one IR and MS are positive and significantly influenced NSMP in the long run. But in the short run, ER had a significant negative influence on NSMP, while MS had a positive and significant effect on NSMP in the short run. In this research, a VAR model is applied to study the effect of MS and ER on IR in Nigeria from January 2010 to December 2019.

METHODS

A secondary data was adopted for this study. The monthly time series IR, MS, and ER data was used and sourced from the Central Bank of Nigeria (CBN). The data spanned from January 2010 to December 2018. The return series of the data was carried out in other to achieve stationarity. To confirm this, the Augmented Dickey–Fuller (ADF) unit root test was performed. In fitting the data, the Vector Autoregressive (VAR) was formulated and implemented. Also, in determining the best fitted model, the Akaike Information Criteria (AIC) and Schwarz Information Criteria (SIC) were used. Also, the impulse response function (IRF) graphs are presented. All of these techniques are presented in the next subsections.

Return series from price

$$\text{Let } R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where, P_t and P_{t-1} are the current and past closing prices, and R_t is the continuous compounded return series which is the natural logarithm of the simple gross return.

Unit root test

Stationarity of the return series is one of the major assumptions in financial time series modelling. This assumption can be checked using the ADF unit root test (Dickey & Fuller, 1997 and Farouk *et al.*, 2021b). Let,

$$x_t = \phi_1 x_{t-1}$$

$$x_t - x_{t-1} = \phi_1 x_t - x_{t-1}$$

$$\Delta x_t = (\phi_i - 1)x_{t-1}$$

$$\Rightarrow \phi_1 - 1 = 0$$

$$\phi_1 = 1.$$

The test of hypothesis is given as;

$$H_0: \phi_1 = 1$$

$$H_1: \phi_1 < 1.$$

The test statistic (t-ratio) is given as;

$$\frac{\phi_1^n - 1}{std(\phi_1)} = \frac{\sum\limits_{t=1}^{S} P_{t-1}e_t}{\sigma_\wedge^2 \sqrt{\sum\limits_{t=1}^{S} P_{t-1}^2}} \qquad (2)$$

where, $\phi_1 = \dfrac{\sum\limits_{t=1}^{S} P_{t-1}P_t}{\sum\limits_{t=1}^{S} P_{t-1}^2}$ and $\hat{\sigma}^2 = \dfrac{\sum\limits_{t=1}^{T}(p_t - \hat{\phi}_1 p_{t-1})^2}{S-1}$.

$P_0 = 0$, S is the sample size, and ϕ_1 for each factors.

The null hypothesis is rejected if the calculated value of t is greater than t critical value (Dikko *et al.*, 2015).

Model Selection Criteria

The AIC and SIC statistics for selecting the best model are presented as follows.

$$\text{AIC} = 2\text{K} - 2\ln(LL) \qquad (3)$$

$$\text{SIC} = K[\log(n)] - 2\ln(LL) \qquad (4)$$

where, K is the number of parameters in the model and LL is the maximized value of the model likelihood function (David *et al.*, 2023).

Vector Autoregressive (VAR) Model

For a set of n time series variables $y_t = (y_{1t}, y_{2t}, ..., y_{nt})'$, a VAR model of order p VAR (p) can be written as:

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + ... + A_p y_{t-p} + u_t. \tag{5}$$

To trace out the time path of the effect of shocks on IR, the VAR is transformed into a Vector Multivariate Analysis (VMA) representation as follows.

The VAR model is more compactly written as;

$$X_t = A_0 + A_1 X_{t-1} + e_t \Rightarrow X_t = \frac{A_0}{I - A_1 L} + \frac{e_t}{I - A_1 L} \tag{6}$$

First, consider the first component on the right hand side:

$$\frac{A_0}{I - A_1} = (I - A_1)^{-1} A_0 = \frac{(I - A_1)^a A_0}{|I - A_1|} = \frac{\begin{bmatrix} 1 - a_{11} & -a_{12} \\ -a_{21} & 1 - a_{22} \end{bmatrix} A_0}{\begin{vmatrix} 1 - a_{11} & -a_{12} \\ -a_{21} & 1 - a_{22} \end{vmatrix}} = \frac{\begin{bmatrix} 1 - a_{22} & a_{21} \\ a_{12} & 1 - a_{22} \end{bmatrix} \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix}}{(1 - a_{11})(1 - a_{22}) - a_{21}a_{12}}$$

$$= \frac{1}{\Delta} \begin{bmatrix} (1 - a_{22})a_{10} + a_{21}a_{20} \\ a_{12}a_{10} + (1 - a_{22})a_{20} \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} \tag{7}$$

Stability requires that the roots of $I - A_1 L$ lie outside the unit circle. In this study the same assumption is followed. Then the second component can be written as:

$$\frac{e_t}{I - A_1 L} = \sum_{i=0}^{\infty} A_1^i e_{t-i} = \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^i \begin{bmatrix} e_{1,t-i} \\ e_{2,t-i} \end{bmatrix} \tag{8}$$

Thus the VAR model can be written as a VMA model with the standard VAR's model error terms.

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{A^i}^i \begin{bmatrix} e_{1,t-i} \\ e_{2,t-i} \end{bmatrix} \tag{9}$$

But these are composite errors consisting of the structural innovations. The e-terms are replaced with the $\varepsilon's$ from (6) as follows;

$$e_t = \frac{1}{|\Delta|} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \varepsilon_t$$

(10)

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \underbrace{\frac{A^i}{1 - b_{12}b_{21}} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix}}_{\Phi_i} \begin{bmatrix} \varepsilon_{y,t-i} \\ \varepsilon_{z,t-i} \end{bmatrix}$$

(11)

$$= \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \Phi_{11}^{(i)} & \Phi_{12}^{(i)} \\ \Phi_{21}^{(i)} & \Phi_{22}^{(i)} \end{bmatrix}^i \begin{bmatrix} \varepsilon_{y,t-i} \\ \varepsilon_{z,t-i} \end{bmatrix}$$

$$= \bar{X} + \sum_{i=0}^{\infty} \Phi_i \varepsilon_{t-i}.$$

(12)

RESULTS

The descriptive statistics for the data are first presented followed by the ADF unit root test results. The next results are the VAR model result, the VAR model validation result, and the IRF graphs.

Table 1. Descriptive Statistics Results

	RTIR	RTMS	RTER
Mean	0.013130	0.020310	0.004345
Median	0.010153	0.018648	0.002475
Maximum	0.077826	0.117204	0.107372
Minimum	-0.020779	-0.044948	-0.047828
Std. Dev.	0.014795	0.026409	0.018352
Skewness	1.041158	0.730426	2.043383
Kurtosis	6.808455	5.662636	14.77512
Jarque-Bera	62.01620	30.36140	511.3775
Probability	0.000000	0.000000	0.000000
Sum	1.037235	1.604452	0.343219
Sum Sq. Dev.	0.017074	0.054400	0.026269
Observations	120	120	120

RTIR~Return of IR. RTMS~Return of MS. RTER~Return of ER

Table 1 shows the descriptive statistic of the study variables which from the output result on the normality:-Jarque-Bera tells us that the series variables are not in normality and the skewness and kurtosis are positive with the return mean, positive.

Table 2. Unit Root Test

Group unit root test: Summary

Series: RTIR, RTMS, RTER

Method	Statistic	Prob.**	Cross Section	Obs.
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-13.0618	0.0000	3	232
ADF - Fisher Chi-square	93.5258	0.0000	3	232
PP - Fisher Chi-square	78.7088	0.0000	3	234

RTIR~Return of IR. RTMS~Return of MS. RTER~Return of ER

Table 2 presents the unit root test and the result shows that the returns of the variables are stationary since the probability value is less than 0.05% and 0.01%. It is therefore concluded that the series is stationary at level. The stationary plot for the three variables are shown in Figure 1.
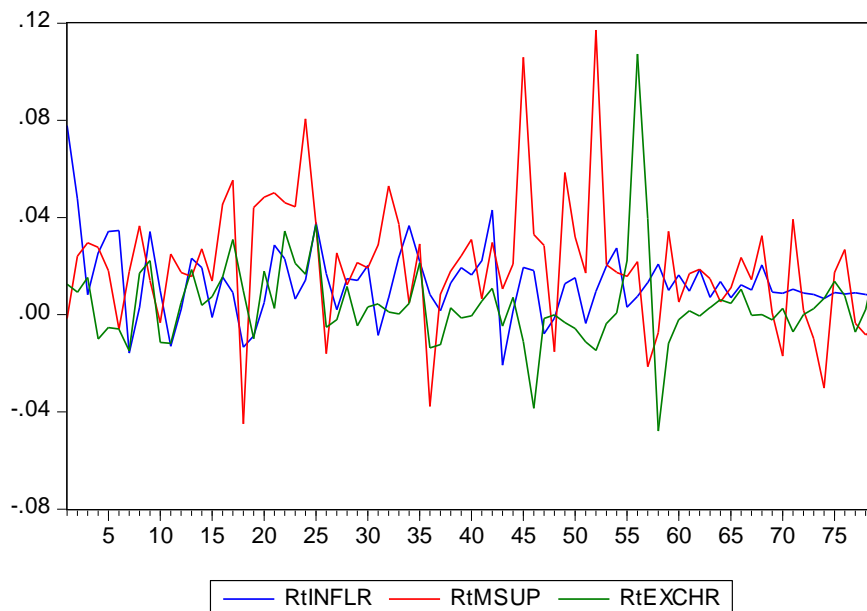

Figure 1. Stationarity Plot for IR, MS, and ER

Table 3. Vector Autoregressions Estimates

Vector Autoregressions Estimates
Sample (adjusted): 3 79
Included observations: 77 after adjustments
Standard errors in ( ) & t-statistic in [ ]

	RTINFLR	RTMSUP	RTEXCHR
RTINFLR(-1)	0.124683	-0.451493	-0.187294
	(0.11117)	(0.24602)	(0.16460)
	[1.12155]	[-1.83516]	[-1.13786]
RTINFLR(-2)	-0.258300	-0.194676	0.183680
	(0.09515)	(0.21058)	(0.14089)
	[-2.71455]	[-0.92448]	[1.30374]
RTMSUP(-1)	0.085905	0.136255	0.043092
	(0.05396)	(0.11942)	(0.07990)
	[1.59191]	[1.14095]	[0.53933]
RTMSUP(-2)	0.028358	0.065135	-0.039623
	(0.05233)	(0.11582)	(0.07749)
	[0.54186]	[0.56239]	[-0.51134]
RTEXCHR(-1)	0.040205	-0.366373	0.399922
	(0.07719)	(0.17082)	(0.11428)
	[0.52089]	[-2.14485]	[3.49938]
RTEXCHR(-2)	2.47E-05	0.198673	-0.298287
	(0.07888)	(0.17457)	(0.11679)
	[0.00031]	[1.13809]	[-2.55396]
C	0.011172	0.025134	0.003618
	(0.00249)	(0.00552)	(0.00369)
	[4.48231]	[4.55680]	[0.98027]
R-squared	0.174199	0.132235	0.200097
Adj. R-squared	0.103416	0.057855	0.131534
Sum sq. Resids	0.009551	0.046775	0.020938
S.E. equation	0.011681	0.025850	0.017295
F-statistic	2.461035	1.777830	2.918434
Log likelihood	237.0471	175.8811	206.8273
Akaike AIC	-5.975249	-4.386521	-5.190319
Schwarz SC	-5.762175	-4.173448	-4.977246
Mean dependent	0.011840	0.020544	0.004174
S.D. dependent	0.012336	0.026632	0.018558
Determinant resid covariance (dof adj.)		2.63E-11	
Determinant resid covariance		1.97E-11	
Log likelihood		621.2226	
Akaike information criterion		-15.59020	
Schwarz criterion		-14.95098	

RTIR~Return of IR. RTMS~Return of MS. RTER~Return of ER

Table 3 shows the results analysis of the VAR (2) model, lag 2 with respective equations of the vector analysis with additional coefficient summary statistic. The bolded and orange colour shows the significance of the coefficient lag.

Table 4. VAR Model Residual Serial Correlation Lagrange Multiplier Test

Null Hypothesis: no serial correlation at lag order h

Sample: 179

Included observations: 77. DF: 9

Lags	LM-Stat	*P*-value
1	11.82620	0.2233
2	17.49849	0.0415
3	5.829542	0.7568

Table 4 shows the validation of after test results of VAR Model which tell us that lag one (1) and lag (3) are not significant at 5% while only lag (2) is significant at 5% but not at 1%.

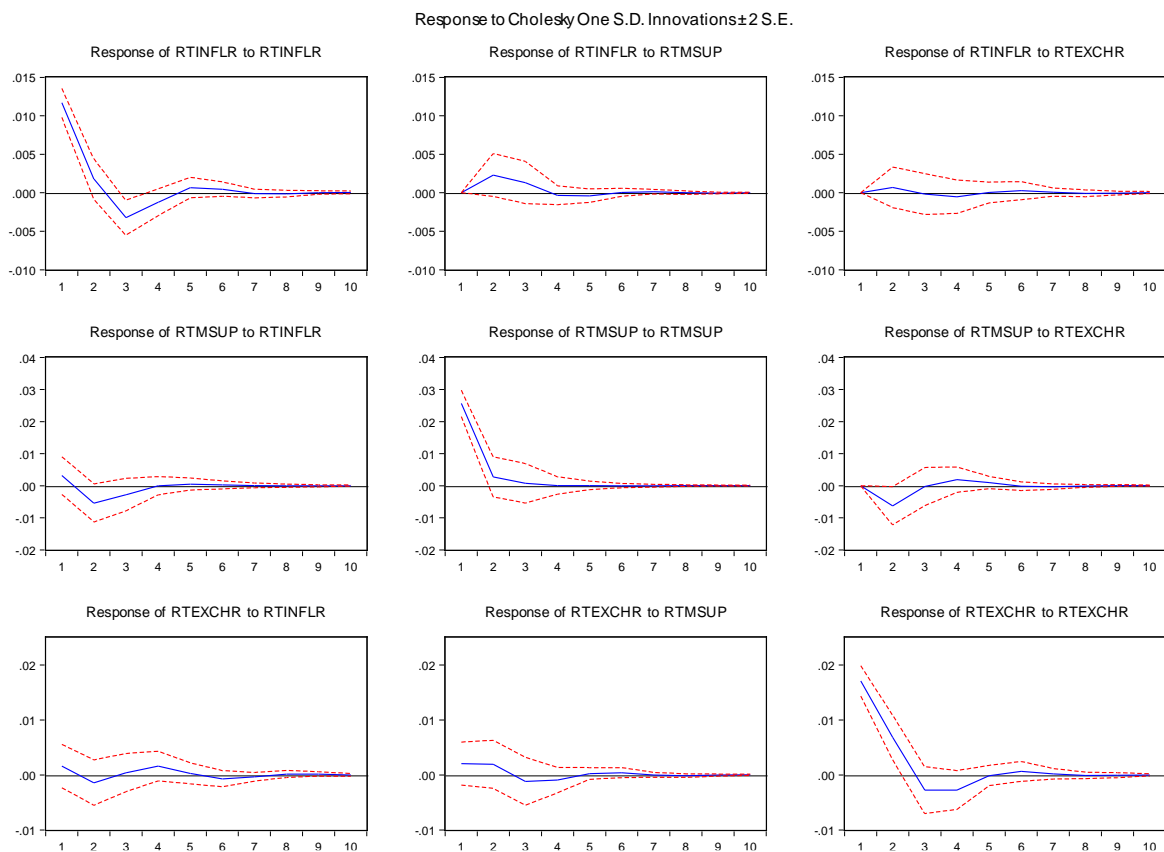Response to Cholesky One S.D. Innovations ± 2 S.E.

Figure 2. Impulse Response Function Graphs

The vertical axis is expressed in units of the dependent variable (Inflation). The solid line is a point estimate for the amount Inflation is expected to change following a unit impulse after the number of periods, as in the case of year(s). The inflation rate in relation to the others variable of the impulse responses appear quite unstable but at the later end of the period shows that the inflation rate in Nigeria will be stabilized.

CONCLUSION

The empirical results, shows the descriptive statistic and stationarity of the return series of the variables with the variables not normally distributed, probability values less than 0.05 and unit root test stationary at level using the ADF (Augmented Dickey–Fuller) test. The results of the VAR model shows some slight significance at different lag were most of the coefficient were not significant and the VAR residual test shows that the VAR model is only significant at lag 2 of the series. The impulse response function graph shows the stability of inflation rate on money supply and exchange rate which appear quite unstable but at later end of the periods shows that inflation rate in Nigeria will be positively stable.

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