

**Effect of Family Role and Law Enforcement Agencies Impact
Analysis of Meth Mathematical Model of Drug Crime
of Users in Nigeria**

Akpienbi Isaac O & Ezra E. T
Federal University Wukari, Nigeria
akpienbi4u@gmail.com

Article Info:

Submitted:	Revised:	Accepted:	Published:
Jun 26, 2024	Jul 12, 2024	Jul 26, 2024	Jul 31, 2024

Abstract

This paper introduces a mathematical model that incorporates family dynamics and law enforcement into the transmission of methamphetamine and drug-related crime. The model's dynamic behavior, equilibrium existence, and global stability are analyzed using mathematical techniques. The research includes the exploration of equilibrium states representing drug crime-free and drug crime-persistent scenarios, as well as the determination of the reproduction number. The findings demonstrate that the drug crime-free equilibrium and drug crime-persistent equilibrium are asymptotically stable. Moreover, the study advises the government to develop and implement policies and strategies aimed at controlling and eradicating drug crime in Nigeria, with a specific focus on law enforcement agencies undertaking the implementation process.

Keywords: Drug Abuse, Drug Crime, Reproduction Number, Law Enforcement Agencies, Stability

Introduction

Drug abuse and its associated criminal activities are pervasive global challenges, and Nigeria is not exempt from this complex issue that spans the entire country (Farai Nyabadza, 2017). The intricate relationship between criminal networks, political and civil institutions safeguarding drug-related activities, and the impact of drug use on the development of criminal governance in Cape Town have been identified (Khalil Goga, 2014). These dynamics have far-reaching negative consequences on families, communities, the environment, and law enforcement agencies, affecting the mental and physical health of drug users and contributing to increased crime rates, traffic accidents, violence, sexual abuse, and injuries (Shakti M, Ramson and Rajendra Chetty, 2016).

The threat of drug abuse extends beyond individual health, significantly impacting the socio-economic fabric of both individuals and nations (T.O, Orwa, and F. Nyabadza, 2019). The consequences of drug abuse, such as divorce, unemployment, and health challenges, are interconnected (J. Mushanyu, 2018). In Cape Town alone, the concentration of substance trader gang memberships is estimated to exceed 100,000, emphasizing the magnitude of the issue (Don Pinnock, 2019). Families and friends play a crucial and desperate role in the lives of drug users and traders (Surapol Naowarat and Nuengruedee Kumat, 2018).

The growth in the supply chains of drugs is directly linked to the increasing number of drug users (Nyabadza et al, 2012). As a response to these challenges, the objective of this study is to propose and analyze a model for the transmission of crime in the context of drug abuse epidemics, focusing on the roles of family and law enforcement agencies.

Model formulation

We consider the total human population of the model at time t , denoted by N , is divided into eight disjoint compartments namely susceptible individuals at risk of using drug S , light drug users I , hard drug users H , clients of health care services in treatment T_{oi} , security forces post investigating crime suspect F_s , suspect on trail for crime C , prison for convicted suspect of crime P , and recovered individuals R . Hence, $N = S + I + H + T_{oi} + F_s + C + P + R$. The model described mathematical system of ordinary differential equation. Where the susceptible individuals is recruited at π rate, the natural dead rate μ ,

β is the per capita contact rate of a product of the effective number of contacts c , between drug users not in treatment and the susceptible population, and the probability $\hat{\beta}$, that a contact results into initiation into drug use, that is $\beta = c\hat{\beta}$, α is the relative infectivity of H when compare to I , r is the level of relapse to being a hard drug user, s is the uptake rate into treatment programs, γ_1 is the recovery rate for light drug users, γ_2 is the recovery rate for drug users under treatment, γ_3 is the recovery rate for drug user and jailed time, β is crime suspect rate, β_1 is the rate at which suspect been release after investigation, η is rate of transfer or charging of suspect to court of law, ϵ is rate at which suspect been discharge when not guilty of the crime, θ_1 is the rate at which suspect been guilty and sentence to prison as well to be held for further hearing of the court, δ is transfer suspect held in prison to court for hearing rate and ν is the effective of the role of family.

Model equation

$$\frac{dS}{dt} = n - \theta SI - \mu S \quad (0.1)$$

$$\frac{dI}{dt} = \theta SI - (\gamma_1 + \alpha + \mu)I \quad (0.2)$$

$$\frac{dH}{dt} = \alpha I + \epsilon C + rR + \beta_1 F_s - (\nu\sigma_1 + \mu + \nu\beta)H \quad (0.3)$$

$$\frac{dT_{oi}}{dt} = \nu\sigma_1 - (\gamma_2 + \mu)T_{oi} \quad (0.4)$$

$$\frac{dF_s}{dt} = \nu\beta - (\beta_1 + \mu + \eta)F_s \quad (0.5)$$

$$\frac{dC}{dt} = \eta F_s + \delta P - (\epsilon + \mu + \theta_1)C \quad (0.6)$$

$$\frac{dP}{dt} = \theta_1 - (\gamma_2 + \delta + \mu)P \quad (0.7)$$

$$\frac{dR}{dt} = \gamma_1 I + \gamma_2 T_{oi} + \gamma_3 P - (r + \mu)R \quad (0.8)$$

With initial condition

$$S(0) = S(0) > 0, I(0) = I(0) \geq 0, H(0) = H(0) \geq 0, T_{oi}(0) = T_{oi}(0) \geq 0,$$

$$F_s(0) = F_s(0) \geq 0, C(0) = C(0) \geq 0, P(0) = P(0) \geq 0, R(0) = R(0) \geq 0$$

Model analysis

Positivity of solution

Now let consider the positivity of crime transmission model of methamphetamine system equations above. We prove that all the state variables remain non negative and the solution of the model system (0.1) - (0.8) with positive initial conditions remain positive for all $t > 0$. The following theorems were considered.

Theorem 1. Given initial conditions of the model system equations (0.1) - (0.8) are

$$S(0) > 0, I(0) > 0, H(0) > 0, T_{oi}(0) > 0, F_s(0) > 0, C(0) > 0, P(0) > 0 \text{ and } R(0) > 0. \text{ there}$$

exists $(S(t), I(t), H(t), T_{oi}(t), F_s(t), C(t), P(t), R(t)): (0, \infty) \rightarrow (0, \infty)$ which solve the model system equations(0.1) - (0.8).

Proof. Assume that

$$\hat{t} = \sup[t > 0 : S > 0, I > 0, H > 0, T_{oi} > 0, F_s > 0, C > 0, P > 0, R > 0] \partial [0, t]. \text{ Thus } \hat{t} > 0, \text{ and}$$

its follows the first equation of the model system (0.1) - (0.8) let $\theta I = \omega$. So that

$$\frac{dS}{dt} = n - (\omega + \mu)S$$

$$S(\hat{t}) \exp\left(\mu\hat{t} + \int_0^{\hat{t}} \omega(s) ds\right) - S(0) \geq 0 \int_0^{\hat{t}} n \exp\left(\mu t + \int_0^t \omega(s) ds\right) dt$$

Then, we have

$$S(\hat{t}) \geq S(0) \exp\left[-\left(\mu\hat{t} + \int_0^{\hat{t}} \omega(s) ds\right)\right] \times \left[\int_0^{\hat{t}} n \exp\left(\mu t + \int_0^t \omega(\tau) d\tau\right) dt\right] > 0.$$

Given

From the second equation of the model system (0.1) - (0.8) we obtain

$$\frac{dI}{dt} = \omega S - (\gamma_1 + \alpha + \mu)I \geq (\gamma_1 + \alpha + \mu)I$$

$$\therefore \Rightarrow I(t) \geq I_0 e^{-(\gamma_1 + \alpha + \mu)t} > 0.$$

In similar manner, it also shown that $H(t) > 0, T_{oi}(t) > 0, F_s(t) > 0, C(t) > 0, P(t) > 0$ and $R(t) > 0$ for all $t > 0$, and the proof is completes

Invariant region

Theorem 2: The feasible region \mathcal{U} defined by

$$\mathcal{U} = \{(S, I, H, T_{oi}, F_s, C, P, R) \in \mathbb{R}_+^8 : 0 \leq N \leq \frac{n}{\mu}\} .$$

With initial conditions

$S_0 \geq 0, I_0 \geq 0, H \geq 0, T_{oi0} \geq 0, F_{s0} \geq 0, C_0 \geq 0, P_0 \geq 0,$ and $R_0 \geq 0$. With respect to the model system (0.1) - (0.8) for all $t > 0$, the \mathcal{U} region is positively invariant.

Proof: Sum up the model equation (0.1) - (0.8); we obtain total population that satisfies

$$\frac{dN}{dt} \leq n - \mu N$$

differential equation that is

We then apply (Birkhoff and Rota) theorem on differential inequalities, and obtain

$$0 \leq N(t) \leq \frac{n}{\mu} + N(0)e^{-\mu t}$$

In equation (0.1) - (0.8), $N(0)$ represents the evaluated value. Taking limit as $t \rightarrow \infty$, we have

$$0 \leq N \leq \frac{n}{\theta} .$$

Indeed, the stated variable in set \mathcal{U} remain biological meaningful.

Drug Reproductive Number (R_{dc})

We use next generation matrix method to determine the drug reproduction number (Van den Driessche and Watmough 2002). The drug reproductive number is defined as the number of secondary infections generated by a typical infected in an otherwise disease free population in whole infectious period. Using matrix notations as in (Van den Driessche and Watmough 2002) the matrices F and V^{-1} are given by

$$F = \begin{bmatrix} 0 \\ \Pi\theta/\mu \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, V =$$

$$\begin{bmatrix} \mu & \Pi\theta/\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (\gamma_1 + \alpha + \mu) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha & (\sigma + v\beta + \mu) & 0 & -\beta_1 & 0 & -\varepsilon & 0 \\ 0 & 0 & -\sigma & (\gamma_2 + \mu) & 0 & 0 & 0 & 0 \\ 0 & 0 & -v\beta & 0 & (\beta_1 + \eta + \mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\eta & (\varepsilon + \mu + \theta_1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\theta_1 & (\gamma_3 + \delta + \mu) & 0 \\ 0 & -\gamma_1 & 0 & -\gamma_2 & 0 & -\gamma_3 & 0 & -\mu \end{bmatrix}$$

And the matrix FV^{-1}

$$FV^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\Pi\theta}{\mu(\gamma_1 + \alpha + \mu)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So, the basic drug reproduction number (R_{dc}) which is the spectral radius of the matrix FV^{-1} is given by

$$R_{dc} = \frac{\Pi\theta}{\mu(\gamma_1 + \alpha + \mu)}$$

Equilibrium points of the model

From the model system equation (1) – (8), by setting the right-hand sides of the equations (1) to zero. We obtained two equilibrium points,

Drug Crime Free Equilibrium (DCFE) denoted by $E_0 = S^*, I, H, T_{oi}, F_s, C, P, R$: In this

case when $I = 0$, we obtained $S = \frac{n}{\mu}, I = 0, H = 0, T_{oi} = 0, F_s = 0, C = 0, P = 0, R = 0$

Drug Crime Persistent Equilibrium (DCPE) denoted by $E_2 = S^*, I^*, H^*, T_{oi}^*, F_s^*, C^*, P^*, R^*$: In the case when $I > 0$, we obtained

From equation (1)

$$S^* = \frac{n}{\theta I^* + \mu} \quad (0.9)$$

Substitute equation (0.9) into equation (0.2)

$$I^* = \frac{-((\gamma_1 + \alpha + \mu)\mu + \theta n) \pm \sqrt{((\gamma_1 + \alpha + \mu)\mu + \theta n)^2}}{(\gamma_1 + \alpha + \mu)\theta} \quad (1.0)$$

From equation (0.4)

$$T_{oi}^* = \frac{v\sigma H^*}{(\gamma_1 + \mu)} \quad (1.1)$$

Substitute equation (1.0) into equation (0.8)

$$R^* = \frac{(\gamma_1 + \mu)\gamma_1 I^* + \gamma_2 v\sigma H^* + (\gamma_1 + \mu)\gamma_3 P^*}{(\gamma_1 + \mu)(r + \mu)} \quad (1.2)$$

Substitute equation (1.2) into equation (0.3)

$$H^* = \frac{(g_3 \alpha + (\gamma_1 + \mu)\gamma_1 r) I^* + g_3 \epsilon C^* + (\gamma_1 + \mu)\gamma_3 r P^* + g_3 \beta_1 F_s^*}{(g_3(\sigma + v\beta + \mu) - \gamma_2 r v \sigma)} \quad (1.3)$$

Substitute (13) into (5)

$$F_s^* = \frac{(g_3 \infty + (\gamma_1 + \mu)\gamma_1 r)v\beta I^* + g_3 \varepsilon v\beta C^* + (\gamma_1 + \mu)\gamma_3 r v\beta P^*}{((\beta_1 + \eta + \mu)g_4 - g_3 \beta_1 v\beta)} \quad (1.4)$$

Substitute (14) into (6)

$$C^* = \frac{(g_3 \infty + (\gamma_1 + \mu)\gamma_1 r)\eta v\beta I^* + ((\gamma_1 + \mu)\gamma_{3\eta} r v\beta + g_5 \delta)P^*}{g_5(\varepsilon + \mu + \theta_1) - g_3 \eta \varepsilon v\beta} \quad (1.5)$$

Put (15) into (7)

$$P^* = \frac{(g_3 \infty + (\gamma_1 + \mu)\gamma_1 r)\eta v\beta \theta_1 I^*}{g_6(\gamma_3 + \delta + \mu) - ((\gamma_1 + \mu)\gamma_{3\eta} r v\beta \theta_1 + g_5 \delta \theta_1)} \quad (1.6)$$

Global stability of the drug crime free steady state

We shall now prove the global stability of the methamphetamine free equilibrium point E_0 whenever the reproduction number is less than unity.

Theorem 3. The methamphetamine free equilibrium point E_0 of model Equations (0.1) - (0.8) is globally *asymptotically stable* for $R_{dc} \leq 1$ and *unstable* if $R_{dc} > 1$.

Proof. Let $L(I, H, T_{oi}, F_s, C, P, R) = w_1 I + w_2 H + w_3 T_{oi} + w_4 F_s + w_5 C + w_6 P + w_7 R$ be a candidate Lyapunov function for some non-negative constants $w_1 - w_7$. The time derivative of L is given by

$$\begin{aligned} \frac{dL}{dt} &= V_1 \frac{dI}{dt} + V_2 \frac{dH}{dt} + V_3 \frac{dT_{oi}}{dt} + V_4 \frac{dF_s}{dt} + V_5 \frac{dC}{dt} + V_6 \frac{dP}{dt} + V_7 \frac{dR}{dt} \\ &\leq V_1 [\theta SI - (\gamma_1 + \infty + \mu)I] + V_2 [\infty I + \varepsilon C + rR + \beta_1 F_s - (v\sigma_1 + \mu + v\beta)H] + V_3 [v\sigma_1 H - (\gamma_2 + \mu)T_{oi}] + \\ &V_4 [v\beta H - (\beta_1 + \mu + \eta)F_s] + V_5 [\eta F_s + \delta P - (\varepsilon + \mu + \theta_1)C] + V_6 [\theta_1 C - (\gamma_2 + \delta + \mu)P] + \\ &V_7 [\gamma_1 I + \gamma_2 T_{oi} + \gamma_3 P - (r + \mu)R] \end{aligned}$$

$$\begin{aligned} &\leq \left[V_1 \left(\frac{\theta n}{\mu} - (\gamma_1 + \alpha + \mu) \right) + V_2 \alpha + V_7 \gamma_1 \right] I + \left[V_3 v \sigma_1 + V_4 v \beta - V_2 (v \sigma_1 + \mu + v \beta) \right] H + (V_7 \gamma_2 - V_3 (\gamma_2 + \mu)) T_{oi} \\ &+ \left[V_2 \beta_1 + V_5 \eta - V_4 (\beta_1 + \mu + \eta) \right] F_s + \left[V_6 \theta_1 + V_2 \varepsilon - V_5 (\varepsilon + \mu + \theta_1) \right] C + \left[V_5 \delta + V_7 \gamma_3 - V_6 (\gamma_2 + \delta + \mu) \right] P \\ &+ \left[V_2 r - V_7 (r + \mu) \right] R \end{aligned}$$

We then evaluate the coefficients of the suitable Lyapunov function such that the coefficients of H, T_{oi}, F_s, C, P and R are equal to zero. We thus obtain

$$= (\gamma_1 + \alpha + \mu) (rv\sigma\gamma_2 g_4 g_5 g_6 - g_2 g_3 g_4 g_5 g_6 g_7 + v\beta\varepsilon\eta g_3 g_6 g_7 + v\beta\beta_1 g_3 g_5 g_6 g_7) (R_{dc} - 1) I$$

Where $g_1 = \gamma_1 + \alpha + \mu, g_2 = \sigma + v\beta + \mu, g_3 = \gamma_2 + \mu, g_4 = \beta_1 + \eta + \mu,$

$$g_5 = \varepsilon + \mu + \theta_1, g_6 = \gamma_3 + \delta + \mu, g_7 = r + \mu.$$

We can deduce that $\frac{dL}{dt} \leq 0$, when $R_{dc} \leq 1$ with equality if $R_{dc} = 0$. Furthermore, $\frac{dL}{dt} = 0$

if and only if $I = H = T_{oi} = F_s = C = P = R = 0$. Therefore, the largest compact invariant set

in $\{(S + I + H + T_{oi} + F_s + C + P + R) \in \square$ such that $\frac{dL}{dt} = 0$ when $R_{dc} \leq 1$ is the singleton

E_0 . By Lasalle invariance principle, this implies that E_0 is globally stable in \square if $R_{dc} \leq 1$.

Global stability of the drug crime persistent equilibrium steady state

In this section, we prove the global asymptotic stability of E_1 by using Lyapunov's direct method. The ideal of (De Leon) were employed in constructing a common quadratic Lyapunov function of the model equations (0.1)-(0.8)

Theorem 4. If $R_{dc} > 1$, the equilibrium point E_1 of model Equations (0.1)-(0.8) is globally asymptotically stable for $R_{dc} \leq 1$ and unstable if $R_{dc} > 1$ in the interior of region \square .

Proof. Let define $L : \{(S, I, H, T_{oi}, F_s, C, P) \in \square : S, I, H, T_{oi}, F_s, C, P > 0\} \rightarrow \mathbb{R}$

Then, we construct common quadratic Lyapunov function of (0.1)-(0.8) and obtain

$$L(S, I, H, T_{oi}, F_s, C, P) = \frac{1}{2} \left[\begin{aligned} &(S - S^*) + (I - I^*) + (H - H^*) + (T_{oi} - T_{oi}^*) + (F_s - F_s^*) + (C - C^*) \\ &+ (P - P^*) \end{aligned} \right]^2 \quad (1.7)$$

L is D_1 in the interior of \square , where E_1 denotes the global minimum of L on \square and $L(S^*, I^*, H^*, T_{oi}^*, F_s^*, C^*, P^*) = 0$

Now, we differentiate L along the solution of the model system equations (0.1)-(0.8), we obtain

$$\begin{aligned} \frac{dL}{dt} &= \left[(S - S^*) + (I - I^*) + (H - H^*) + (T_{oi} - T_{oi}^*) + (F_s - F_s^*) + (C - C^*) + (P - P^*) \right] \frac{d}{dt} \\ &(S + I + H + T_{oi} + F_s + C + P) \end{aligned} \quad (1.8)$$

Where

$$\frac{d}{dt}(S + I + H + T_{oi} + F_s + C + P) = n - \mu(S + I + H + T_{oi} + F_s + C + P) - \gamma_1 I - \gamma_2 T_{oi} - \gamma_3 P$$

Then, equations (0.1)-(0.8) becomes

$$\begin{aligned} \frac{dL}{dt} &= \left[(S - S^*) + (I - I^*) + (H - H^*) + (T_{oi} - T_{oi}^*) + (F_s - F_s^*) + (C - C^*) + (P - P^*) \right] \times n \\ &- \mu(S + I + H + T_{oi} + F_s + C + P) - \gamma_1 I - \gamma_2 T_{oi} - \gamma_3 P \end{aligned} \quad (1.9)$$

At the equilibrium point E_1 , Π in equation (1.9) becomes

$$n = \mu(S^* + I^* + H^* + T_{oi}^* + F_s^* + C^* + P^*) + \gamma_1 I^* + \gamma_2 T_{oi}^* + \gamma_3 P^*$$

Therefore, equations (1.9) can be rewritten as

$$\begin{aligned} \frac{dL}{dt} &= \left[(S - S^*) + (I - I^*) + (H - H^*) + (T_{oi} - T_{oi}^*) + (F_s - F_s^*) + (C - C^*) + (P - P^*) \right] \\ &\times \left(\begin{aligned} &\mu(S^* + I^* + H^* + T_{oi}^* + F_s^* + C^* + P^*) - \mu(S + I + H + T_{oi} + F_s + C + P) + \gamma_1 I^* + \gamma_2 T_{oi}^* + \gamma_3 P^* \\ &+ \gamma_1 I + \gamma_2 T_{oi} + \gamma_3 P \end{aligned} \right) \end{aligned} \quad (2.0)$$

From equations (2.0) we have

$$\frac{dL}{dt} = \left[(S - S^*) + (I - I^*) + (H - H^*) + (T_{oi} - T_{oi}^*) + (F_s - F_s^*) + (C - C^*) + (P - P^*) \right] \times$$

$$\left(\mu \left[(S - S^*) + (I - I^*) + (H - H^*) + (T_{oi} - T_{oi}^*) + (F_s - F_s^*) + (C - C^*) + (P - P^*) \right] - \right.$$

$$\left. \left[\gamma_1 (I - I^*) - \gamma_2 (T_{oi} - T_{oi}^*) - \gamma_3 (P - P^*) \right] \right)$$

(2.1)

Let

$$M_1 = (S - S^*), M_2 = (I - I^*), M_3 = (H - H^*), M_4 = (T_{oi} - T_{oi}^*), M_5 = (F_s - F_s^*), M_6 = (C - C^*),$$

$$M_7 = (P - P^*), M_8 = M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7$$

Thus, equation (2.1) becomes

$$\frac{dL}{dt} = M_8 \left[-\mu M_8 - \gamma_1 M_2 - \gamma_2 M_4 - \gamma_3 M_7 \right]$$

It can also be rewritten as

$$\frac{dL}{dt} = - \left[\mu M_8^2 + \gamma_1 M_2 M_8 + \gamma_2 M_4 M_8 + \gamma_3 M_7 M_8 \right]$$

Therefore,

$$\frac{dL}{dt} = - \left[\mu M_8^2 + \gamma_1 M_2 M_8 + \gamma_2 M_4 M_8 + \gamma_3 M_7 M_8 \right] \leq 0$$

Also, $\frac{dL}{dt} = 0$ if $S = S^*, I = I^*, H = H^*, T_{oi} = T_{oi}^*, F_s = F_s^*, C = C^*$ and $P = P^*$ in equation

(2.1). So, the largest compact invariant set in $(S, I, H, T_{oi}, F_s, C, P)$ is singleton E_1 , where E_1 is the endemic equilibrium point. By Lassalle's invariance principle E_1 is globally asymptotically stable in the interior of \square .

E_1 , where E_1 is the endemic equilibrium point. By Lassalle's invariance principle E_1 is globally asymptotically stable in the interior of \square .

Conclusion

The study delved into a tentative examination of a model concerning the transmission of methamphetamine-related crimes. The analysis revealed that the model's dynamic behavior is contingent upon the basic reproduction number R_{dc} . When $R_{dc} < 1$, the drug crime-free equilibrium is asymptotically stable, indicating that the issue will gradually fade away. This

occurs when instances of drug infections are identified through familial roles and effectively treated by healthcare centers and law enforcement agencies before escalating to active drug use or trading. On the contrary, when $R_{dc} > 1$, the drug crime-free equilibrium becomes unstable, leading to the persistence of drug-related crimes. Furthermore, the study demonstrated that the endemic drug crime equilibrium achieves global asymptotic stability when the reproduction number is below unity. Consequently, the study agrees with Akpienbi *et al.* (2021) for implementing policies and control strategies become crucial in eliminating drug abuse, aiming to bring the reproduction number below unity. This approach not only addresses the issue at its root but also serves to reduce the threshold number necessary for sustaining drug-related crimes.

References

1. Driessche P V, and J Watmough J 2002 *Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission*. Math Biosci, 2002(180), pp29-48.
2. Akpienbi I. O., Shiaki Bulus O., Haruna B., Ogwumu O. D. and Nwaokolo M. O. (2021). The Seir Mathematical Modeling of Drug Infection Transmission in Northern Region of Taraba State, Nigeria. *Asian Journal of Pure and Applied Mathematics* 3(2), 34-44
3. Farai Nyabadza and Lezanic Coetzee 2017 *A systems dynamic model for drug abuse and drug related crime in the Western Cape Province of South Africa*. Hindawi computational and mathematical methods in Medicine Vol. 2017(13)
4. James F. Quinn and Zach Sneed 2018 *drug and crime: An empirical based, interdisciplinary model*. Journal of teaching in addictions Vol. 7(1)
5. JP LaSalle. The stability of dynamical systems, regional conference series in appl. Math., SLAM, Philadelphia, 1976.
6. J. Mushanyu 2018 *Analysis of a drug abuse model with a piecewise defined treatment function*. World Journal of Modelling and Simulation Vol. 15 (2019) No. 3, pp. 170-186
7. Don Pinnock 2019 *Criminalization key to Cape Town's drug war*. Centre for criminology, University of Cape Town.
8. Khalil Goga 2014. *The drug trade and governance in Cape Town*. Institute for security studies 2014.
9. Nyabadza, F. N. Jagarah, J. B. H. and Smith, R. J. 2012. *Modelling the dynamic of crystal meth ('Tik') abuse in the presence of drug-supply chains in South Africa*. Bull. Math. 2012(1) pp 24-48
10. Surapol Naowarat and Nuengruedee Kumat 2018 *The family role on transmission model of methamphetamine*. Journal of physics: Conf. Series 1039(2018)012036.
11. Shakti M. Ramson and Rajendra Chetty 2016 Taking strain theorizing drug use in the cape flats. Acta criminological southern African journal of Criminology 29(3).
12. T.O. Orwa and F Nyabadza 2019. *Mathematical modeling and analysis of alcohol-methamphetamine co-abuse in the Western Cape Province of South Africa*. Cogent mathematics and statistics