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Effect of Family Role and Law Enforcement Agencies Impact Analysis of Meth Mathematical Model of Drug Crime of Users in Nigeria

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Abstract

This paper introduces a mathematical model that incorporates family dynamics and law enforcement into the transmission of methamphetamine and drug-related crime. The model's dynamic behavior, equilibrium existence, and global stability are analyzed using mathematical techniques. The research includes the exploration of equilibrium states representing drug crime-free and drug crime-persistent scenarios, as well as the determination of the reproduction number. The findings demonstrate that the drug crime-free equilibrium and drug crime-persistent equilibrium are asymptotically stable. Moreover, the study advises the government to develop and implement policies and strategies aimed at controlling and eradicating drug crime in Nigeria, with a specific focus on law enforcement agencies undertaking the implementation process.

Keywords: Drug Abuse, Drug Crime, Reproduction Number, Law Enforcement Agencies, Stability

Introduction

Drug abuse and its associated criminal activities are pervasive global challenges, and Nigeria is not exempt from this complex issue that spans the entire country (Farai Nyabadza, 2017). The intricate relationship between criminal networks, political and civil institutions safeguarding drug-related activities, and the impact of drug use on the development of criminal governance in Cape Town have been identified (Khalil Goga, 2014). These dynamics have far-reaching negative consequences on families, communities, the environment, and law enforcement agencies, affecting the mental and physical health of drug users and contributing to increased crime rates, traffic accidents, violence, sexual abuse, and injuries (Shakti M, Ramson and Rajendra Chetty, 2016).

The threat of drug abuse extends beyond individual health, significantly impacting the socio-economic fabric of both individuals and nations (T.O, Orwa, and F. Nyabadza, 2019). The consequences of drug abuse, such as divorce, unemployment, and health challenges, are interconnected (J. Mushanyu, 2018). In Cape Town alone, the concentration of substance trader gang memberships is estimated to exceed 100,000, emphasizing the magnitude of the issue (Don Pinnock, 2019). Families and friends play a crucial and desperate role in the lives of drug users and traders (Surapol Naowarat and Nuengruedee Kumat, 2018).

The growth in the supply chains of drugs is directly linked to the increasing number of drug users (Nyabadza et al, 2012). As a response to these challenges, the objective of this study is to propose and analyze a model for the transmission of crime in the context of drug abuse epidemics, focusing on the roles of family and law enforcement agencies.

Model formulation

We consider the total human population of the model at time t, denoted by N, is divided into eight disjoint compartments namely susceptible individuals at risk of using drug S, light drug users I, hard drug users H, clients of health care services in treatment T_{oi}, security forces post investigating crime suspect F_s, suspect on trail for crime C, prison for convicted suspect of crime P, and recovered individuals R. Hence, $N = S + I + H + T_{oi} + F_s + C + P + R$. The model described mathematical system of ordinary differential equation. Where the susceptible individuals is recruited at π rate, the natural dead rate μ,

β is the per capita contact rate of a product of the effective number of contacts c, between drug users not in treatment and the susceptible population, and the probability$\hat{\beta}$, that a contact results into initiation into drug use, that is $\beta = c\hat{\beta}$, α is the relative infectivity of $H$when compare to I , r is the level of relapse to being a hard drug user, s is the uptake rate into treatment programs, γ_1 is the recovery rate for light drug users, γ_2 is the recovery rate for drug users under treatment, γ_3 is the recovery rate for drug user and jailed time, β is crime suspect rate, β_1 is the rate at which suspect been release after investigation, η is rate of transfer or charging of suspect to court of law, ε is rate at which suspect been discharge when not guilty of the crime, $\theta_1$is the rate at which suspect been guilty and sentence to prison as well to be held for further hearing of the court, δ is transfer suspect held in prison to court for hearing rate and v is the effective of the role of family.

Model equation

$$\frac{dS}{dt} = n - \theta SI - \mu S \tag{0.1}$$

$$\frac{dI}{dt} = \theta SI - \left(\gamma_1 + \propto + \mu\right) I \tag{0.2}$$

$$\frac{dH}{dt} = \propto I + \varepsilon C + rR + \beta_1 F_s - \left(v\sigma_1 + \mu + v\beta\right) H \tag{0.3}$$

$$\frac{dT_{oi}}{dt} = v\sigma_1 - \left(\gamma_2 + \mu\right) T_{oi} \tag{0.4}$$

$$\frac{dF_s}{dt} = v\beta - \left(\beta_1 + \mu + \eta\right) F_s \tag{0.5}$$

$$\frac{dC}{dt} = \eta F_s + \delta P - \left(\varepsilon + \mu + \theta_1\right) C \tag{0.6}$$

$$\frac{dP}{dt} = \theta_1 - \left(\gamma_2 + \delta + \mu\right) P \tag{0.7}$$

$$\frac{dR}{dt} = \gamma_1 I + \gamma_2 T_{oi} + \gamma_3 P - \left(r + \mu\right) R \tag{0.8}$$

With initial condition

$$S\left(0\right) = S\left(0\right) > 0, I\left(0\right) = I\left(0\right) \geq 0, H\left(0\right) = H\left(0\right) \geq 0, T_{oi}\left(0\right) = T_{oi}\left(0\right) \geq 0,$$

$$F_s(0) = F_s(0) \geq 0, C(0) = C(0) \geq 0, P(0) = P(0) \geq 0, R(0) = R(0) \geq 0$$

Model analysis

Positivity of solution

Now let consider the positivity of crime transmission model of methamphetamine system equations above. We prove that all the state variables remain non negative and the solution of the model system (0.1) - (0.8) with positive initial conditions remain positive for all $t > 0$. The following theorems were considered.

Theorem 1. Given initial conditions of the model system equations (0.1) - (0.8) are $S(0) > 0, I(0) > 0, H(0) > 0, T_{oi}(0) > 0, F_s(0) > 0, C(0) > 0, P(0) > 0$ and $R(0) > 0.$ there exists $(S(t), I(t), H(t), T_{oi}(t), F_s(t), C(t), P(t), R(t)) : (0, \infty) \to (0, \infty)$ which solve the model system equations(0.1) - (0.8).

Proof. Assume that

$\hat{t} = \sup[t > 0 : S > 0, I > H > 0, T_{oi} > 0, F_s > 0, C > 0, P > 0, R > 0] \partial [0, t]$ .Thus $\hat{t} > 0$, and its follows the first equation of the model system (0.1) - (0.8) let $\theta I = \omega$. So that

$$\frac{dS}{dt} = n - (\omega + \mu) S$$

$$S\left(\hat{t}\right) \exp\left(\mu \hat{t} + \int_0^{\hat{t}} \omega(s) ds\right) - S(0) \geq 0 \int_0^{\hat{t}} n \exp\left(\mu \hat{t} + \int_0^{\hat{t}} \omega(s) ds\right) d\hat{t}$$

Then, we have

$$S\left(\hat{t}\right) \geq S(0) \exp\left[-\left(\mu \hat{t} + \int_0^{\hat{t}} \omega(s) ds\right)\right] \times \left[\int_0^{\hat{t}} n \exp\left(\mu \hat{t} + \int_0^{\hat{t}} \omega(\tau) d\tau\right) d\hat{t}\right] > 0.$$

Given

From the second equation of the model system (0.1) - (0.8) we obtain

$$\frac{dI}{dt} = \omega S - (\gamma_1 + \propto + \mu) I \geq (\gamma_1 + \propto + \mu) I$$

$$\therefore \Rightarrow I(\hat{t}) \geq I_0 e^{-(\gamma_1 + \propto + \mu)\hat{t}} > 0.$$

In similar manner, it also shown that $H(t)>0, T_{oi}(t)>0, F_s(t)>0, C(t)>0, P(t)>0$ and $R(t)>0$ for all $t>0$, and the proof is completes

Invariant region

Theorem 2: The feasible region defined by

 $=\{(S,I,H,T_{oi},F_s,C,P,R)\in \mathrm{R}_+^{\,8}: 0\le N\le \frac{n}{\mu}\}$. With initial conditions $S_0\ge 0, I_0\ge 0, H\ge 0, T_{oi0}\ge 0, F_{s0}\ge 0, C_0\ge 0, P_0\ge 0,$ and $R_0\ge 0$. With respect to the model system (0.1) - (0.8) for all t > 0, the υ region is positively invariant.

Proof: Sum up the model equation (0.1) - (0.8); we obtain total population that satisfies differential equation that is $\frac{dN}{dt}\le nn-\mu N$

We then apply (Birkhoff and Rota) theorem on differential inequalities, and obtain

$0\le N(t)\le \frac{n}{\mu}+N(0)e^{-\mu t}$.

In equation (0.1) - (0.8), $N(0)$ represents the evaluated value. Taking limit as $t\to\infty$, we have

$0\le N\le \frac{n}{\theta}$. Indeed, the stated variable in set υ remain biological meaningful.

Drug Reproductive Number (R_{dc})

We use next generation matrix method to determine the drug reproduction number (Van den Driessche and Watmough 2002). The drug reproductive number is defined as the number of secondary infections generated by a typical infected in an otherwise disease free population in whole infectious period. Using matrix notations as in (Van den Driessche and Watmough 2002) the matrices F and V^{-1} are given by

$$F = \begin{bmatrix} 0 \\ \Pi\theta/\mu \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad ,V =$$

$$\begin{bmatrix} \mu & \Pi\theta/\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (\gamma_1+\alpha+\mu) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha & (\sigma+\nu\beta+\mu) & 0 & -\beta_1 & 0 & -\varepsilon & 0 \\ 0 & 0 & -\sigma & (\gamma_2+\mu) & 0 & 0 & 0 & 0 \\ 0 & 0 & -\nu\beta & 0 & (\beta_1+\eta+\mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\eta & (\varepsilon+\mu+\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\theta_1 & (\gamma_3+\delta+\mu) & 0 \\ 0 & -\gamma_1 & 0 & -\gamma_2 & 0 & -\gamma_3 & 0 & -\mu \end{bmatrix}$$

And the matrix FV^{-1}

$$FV^{-1} = \begin{bmatrix} 0 & \frac{\Pi\theta}{\mu(\gamma_1+\alpha+\mu)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So, the basic drug reproduction number (R_{dc}) which is the spectral radius of the matrix FV^{-1} is given by

$$R_{dc} = \frac{\Pi\theta}{\mu(\gamma_1+\alpha+\mu)}$$

Equilibrium points of the model

From the model system equation $(1)-(8)$, by setting the right-hand sides of the equations (1) to zero. We obtained two equilibrium points,

Drug Crime Free Equilibrium (DCFE) denoted by $E_0 = S^*, I, H, T_{oi}, F_s, C, P, R$ **:** In this case when 1 $I = 0$, we obtained $S = \frac{n}{\mu}, I = 0, H = 0, T_{oi} = 0, F_s = 0, C = 0, P = 0, R = 0$

Drug Crime Persistent Equilibrium (DCPE) denoted by $E_2 = S^*, I^*, H^*, T_{oi}{}^*, F_s^*, C^*, P^*, R^*$ **:** In the case when $I > 0$, we obtained

From equation (1)

$$S^* = \frac{n}{\theta I^* + \mu} \quad (0.9)$$

Substitute equation (0.9) into equation (0.2)

$$I^* = \frac{-\left(\left(\gamma_1 + \alpha + \mu\right)\mu + \theta n\right) \pm \sqrt{\left(\left(\gamma_1 + \alpha + \mu\right)\mu + \theta n\right)}}{\left(\gamma_1 + \alpha + \mu\right)\theta} \quad (1.0)$$

From equation (0.4)

$$T_{oi}{}^* = \frac{v\sigma H^*}{\left(\gamma_1 + \mu\right)} \quad (1.1)$$

Substitute equation (1.0) into equation (0.8)

$$R^* = \frac{\left(\gamma_1 + \mu\right)\gamma_1 I^* + \gamma_2 v\sigma H^* + \left(\gamma_1 + \mu\right)\gamma_3 P^*}{\left(\gamma_1 + \mu\right)\left(r + \mu\right)} \quad (1.2)$$

Substitute equation (1.2) into equation (0.3)

$$H^* = \frac{\left(g_3 \propto + \left(\gamma_1 + \mu\right)\gamma_1 r\right) I^* + g_3 \varepsilon C^* + \left(\gamma_1 + \mu\right)\gamma_3 r P^* + g_3 \beta_1 F_s^*}{\left(g_3\left(\sigma + v\beta + \mu\right) - \gamma_2 rv\sigma\right)} \quad (1.3)$$

Substitute (13) into (5)

$$F_s^* = \frac{\left(g_3 \propto +\left(\gamma_1+\mu\right)\gamma_1 r\right)\nu\beta I^* + g_3\varepsilon\nu\beta C^* + \left(\gamma_1+\mu\right)\gamma_3 r\nu\beta P^*}{\left(\left(\beta_1+\eta+\mu\right)g_4 - g_3\beta_1\nu\beta\right)} \tag{1.4}$$

Substitute (14) into (6)

$$C^* = \frac{\left(g_3 \propto +\left(\gamma_1+\mu\right)\gamma_1 r\right)\eta\nu\beta I^* + \left(\left(\gamma_1+\mu\right)\gamma_{3\eta} r\nu\beta + g_5\delta\right)P^*}{g_5\left(\varepsilon+\mu+\theta_1\right) - g_3\eta\varepsilon\nu\beta} \tag{1.5}$$

Put (15) into (7)

$$P^* = \frac{\left(g_3 \propto +\left(\gamma_1+\mu\right)\gamma_1 r\right)\eta\nu\beta\theta_1 I^*}{g_6\left(\gamma_3+\delta+\mu\right) - \left(\left(\gamma_1+\mu\right)\gamma_{3\eta} r\nu\beta\theta_1 + g_5\delta\theta_1\right)} \tag{1.6}$$

Global stability of the drug crime free steady state

We shall now prove the global stability of the methamphetamine free equilibrium point E_0 whenever the reproduction number is less than unity.

***Theorem* 3.** The methamphetamine free equilibrium point E_0 of model Equations (0.1) - (0.8) is globally *asymptotically stable for* $R_{dc} \le 1$ *and unstable if* $R_{dc} > 1$.

Proof. Let $L\left(I, H, T_{oi}, F_s, C, P, R\right) = w_1 I + w_2 H + w_3 T_{oi} + w_4 F_s + w_5 C + w_6 P + w_7 R$ be a candidate Lyapunov function for some non-negative constants $w_1 - w_7$. The time derivative of L is given by

$$\frac{dL}{dt} = V_1\frac{dI}{dt} + V_2\frac{dH}{dt} + V_3\frac{dT_{oi}}{dt} + V_4\frac{dF_s}{dt} + V_5\frac{dC}{dt} + V_6\frac{dP}{dt} + V_7\frac{dR}{dt}$$

$$\le V_1\left[\theta SI - \left(\gamma_1 + \propto +\mu\right)I\right] + V_2\left[\propto I + \varepsilon C + rR + \beta_1 F_s - \left(\nu\sigma_1 + \mu + \nu\beta\right)H\right] + V_3\left[\nu\sigma_1 H - \left(\gamma_2+\mu\right)T_{oi}\right] +$$
$$V_4\left[\nu\beta H - \left(\beta_1+\mu+\eta\right)F_s\right] + V_5\left[\eta F_s + \delta P - \left(\varepsilon+\mu+\theta_1\right)C\right] + V_6\left[\theta_1 C - \left(\gamma_2+\delta+\mu\right)P\right] +$$
$$V_7\left[\gamma_1 I + \gamma_2 T_{oi} + \gamma_3 P - \left(r+\mu\right)R\right]$$

$$\leq \left[V_1\left(\frac{\theta n}{\mu} - \left(\gamma_1 + \propto + \mu\right)\right) + V_2 \propto + V_7\gamma_1 \right] I + \left[V_3 \nu\sigma_1 + V_4 \nu\beta - V_2\left(\nu\sigma_1 + \mu + \nu\beta\right)\right] H + \left(V_7\gamma_2 - V_3\left(\gamma_2 + \mu\right)\right) T_{oi}$$
$$+ \left[V_2\beta_1 + V_5\eta - V_4\left(\beta_1 + \mu + \eta\right)\right] F_s + \left[V_6\theta_1 + V_2\varepsilon - V_5\left(\varepsilon + \mu + \theta_1\right)\right] C + \left[V_5\delta + V_7\gamma_3 - V_6\left(\gamma_2 + \delta + \mu\right)\right] P$$
$$+ \left[V_2 r - V_7\left(r + \mu\right)\right] R$$

We then evaluate the coefficients of the suitable Lyapunov function such that the coefficients of H, T_{oi}, F_s, C, P and R are equal to zero. We thus obtain

$$= \left(\gamma_1 + \propto + \mu\right)\left(r\nu\sigma\gamma_2 g_4 g_5 g_6 - g_2 g_3 g_4 g_5 g_6 g_7 + \nu\beta\varepsilon\eta g_3 g_6 g_7 + \nu\beta\beta_1 g_3 g_5 g_6 g_7\right)\left(R_{dc} - 1\right) I$$

Where $g_1 = \gamma_1 + \alpha + \mu, g_2 = \sigma + \nu\beta + \mu, g_3 = \gamma_2 + \mu, g_4 = \beta_1 + \eta + \mu,$

$g_5 = \varepsilon + \mu + \theta_1, g_6 = \gamma_3 + \delta + \mu, g_7 = r + \mu$.

We can deduce that $\frac{dL}{dt} \leq 0$, when $R_{dc} \leq 1$ with equality if $R_{dc} = 0$. Furthermore, $\frac{dL}{dt} = 0$ if and only if $I = H = T_{oi} = F_s = C = P = R = 0$. Therefore, the largest compact invariant set in $\{(S + I + H + T_{oi} + F_s + C + P + R) \in$ such that $\frac{dL}{dt} = 0$ when $R_{dc} \leq 1$ is the singleton E_0. By Lasalle invariance principle, this implies that E_0 is globally stable in if $R_{dc} \leq 1$.

Global stability of the drug crime persistent equilibrium steady state

In this section, we prove the global asymptotic stability of E_1 by using Lyapunov's direct method. The ideal of (De Leon) were employed in constructing a common quadratic Lyapunov function of the model equations (0.1)-(0.8)

***Theorem* 4.** If $\mathrm{R_{dc}} > 1$, the equilibrium point E_1 of model Equations (0.1)-(0.8) is globally asymptotically stable for $\mathrm{R_{dc}} \leq 1$ and unstable if $\mathrm{R_{dc}} > 1$ in the interior of region .

Proof. Let define $L : \left\{\left(S, I, H, T_{oi}, F_s, C, P\right) \in : S, I, H, T_{oi}, F_s, C, P > 0\right\} \to \mathrm{R}$

Then, we construct common quadratic Lyapunov function of (0.1)-(0.8) and obtain

$$L(S,I,H,T_{oi},F_s,C,P)=\frac{1}{2}\left[\begin{matrix}(S-S^*)+(I-I^*)+(H-H^*)+(T_{oi}-T_{oi}^*)+(F_s-F_s^*)+(C-C^*)\\+(P-P^*)\end{matrix}\right]^2 \quad (1.7)$$

L is D_1 in the interior of □ , where E_1 denotes the global minimum of L on □ hand $L(S^*,I^*,H^*,T_{oi}^*,F_s^*,C^*,P^*)=0$

Now, we differentiate L along the solution of the model system equations (0.1)-(0.8), we obtain

$$\frac{dL}{dt}=\left[(S-S^*)+(I-I^*)+(H-H^*)+(T_{oi}-T_{oi}^*)+(F_s-F_s^*)+(C-C^*)+(P-P^*)\right]\frac{d}{dt}$$
$$(S+I+H+T_{oi}+F_s+C+P)$$

(1.8)

Where

$$\frac{d}{dt}(S+I+H+T_{oi}+F_s+C+P)=n-\mu(S+I+H+T_{oi}+F_s+C+P)-\gamma_1 I-\gamma_2 T_{oi}-\gamma_3 P$$

Then, equations (0.1)-(0.8) becomes

$$\frac{dL}{dt}=\left[(S-S^*)+(I-I^*)+(H-H^*)+(T_{oi}-T_{oi}^*)+(F_s-F_s^*)+(C-C^*)+(P-P^*)\right]\times n$$
$$-\mu(S+I+H+T_{oi}+F_s+C+P)-\gamma_1 I-\gamma_2 T_{oi}-\gamma_3 P$$

(1.9)

At the equilibrium point E_1, п in equation (1.9) becomes

$$n=\mu(S^*+I^*+H^*+T_{oi}^*+F_s^*+C^*+P^*)+\gamma_1 I^*+\gamma_2 T_{oi}^*+\gamma_3 P^*$$

Therefore, equations (1.9) can be rewritten as

$$\frac{dL}{dt}=\left[(S-S^*)+(I-I^*)+(H-H^*)+(T_{oi}-T_{oi}^*)+(F_s-F_s^*)+(C-C^*)+(P-P^*)\right]$$
$$\times\left(\begin{matrix}\mu(S^*+I^*+H^*+T_{oi}^*+F_s^*+C^*+P^*)-\mu(S+I+H+T_{oi}+F_s+C+P)+\gamma_1 I^*+\gamma_2 T_{oi}^*+\gamma_3 P^*\\+\gamma_1 I+\gamma_2 T_{oi}+\gamma_3 P\end{matrix}\right) \quad (2.0)$$

From equations (2.0) we have

$$\frac{dL}{dt}=\left[\left(S-S^*\right)+\left(I-I^*\right)+\left(H-H^*\right)+\left(T_{oi}-T_{oi}^*\right)+\left(F_s-F_s^*\right)+\left(C-C^*\right)+\left(P-P^*\right)\right]\times$$
$$\left(\begin{array}{l}\mu\left[\left(S-S^*\right)+\left(I-I^*\right)+\left(H-H^*\right)+\left(T_{oi}-T_{oi}^*\right)+\left(F_s-F_s^*\right)+\left(C-C^*\right)+\left(P-P^*\right)\right]-\\ \gamma_1\left(I-I^*\right)-\gamma_2\left(T_{oi}-T_{oi}^*\right)-\gamma_3\left(P-P^*\right)\end{array}\right)$$
(2.1)

Let

$$M_1=\left(S-S^*\right),M_2=\left(I-I^*\right),M_3=\left(H-H^*\right),M_4=\left(T_{oi}-T_{oi}^*\right),M_5=\left(F_s-F_s^*\right),M_6=\left(C-C^*\right),$$
$$M_7=\left(P-P^*\right),M_8=M_1+M_2+M_3+M_4+M_5+M_6+M_7$$

Thus, equation (2.1) becomes

$$\frac{dL}{dt}=M_8\left[-\mu M_8-\gamma_1M_2-\gamma_2M_4-\gamma_3M_7\right]$$

It can also be rewritten as

$$\frac{dL}{dt}=-\left[\mu M_8^{\,2}+\gamma_1M_2M_8+\gamma_2M_4M_8+\gamma_3M_7M_8\right]$$

Therefore,

$$\frac{dL}{dt}=-\left[\mu M_8^{\,2}+\gamma_1M_2M_8+\gamma_2M_4M_8+\gamma_3M_7M_8\right]\le 0$$

Also, $\frac{dL}{dt}=0$ if $S=S^*,I=I^*,H=H^*,T_{oi}=T_{oi}^*,F_s=F_s^*,C=C^*$ and $P=P^*$ in equation (2.1). So, the largest compact invariant set in $\left(S,I,H,T_{oi},F_s,C,P\right)\partial\square:\frac{dL}{dt}=0$ is singleton E_1, where E_1 is the endemic equilibrium point. By Lassalle's invariance principle E_1 is globally asymptotically stable in the interior of $\square$.

Conclusion

The study delved into a tentative examination of a model concerning the transmission of methamphetamine-related crimes. The analysis revealed that the model's dynamic behavior is contingent upon the basic reproduction number R_{dc}. When $R_{dc}<1$, the drug crime-free equilibrium is asymptotically stable, indicating that the issue will gradually fade away. This

occurs when instances of drug infections are identified through familial roles and effectively treated by healthcare centers and law enforcement agencies before escalating to active drug use or trading. On the contrary, when $R_{dc} > 1$, the drug crime-free equilibrium becomes unstable, leading to the persistence of drug-related crimes. Furthermore, the study demonstrated that the endemic drug crime equilibrium achieves global asymptotic stability when the reproduction number is below unity. Consequently, the study agrees with Akpienbi *et al.* (2021) for implementing policies and control strategies become crucial in eliminating drug abuse, aiming to bring the reproduction number below unity. This approach not only addresses the issue at its root but also serves to reduce the threshold number necessary for sustaining drug-related crimes.

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