African Multidisciplinary
Journal of Sciences and Artificial Intelligence

ISSN : 0000-0000

Index: Harvard, Boston, Sydney University, Dimensions, Lens, ResearchGet Scilit, Semantic, Google Scholar, Base etc

https://doi.org/10.58578/AMJSAI.v1i1.3389

Mathematical Modelling of Crime Rate and Prison Population in Nigeria

Okorie Charity Ebelechukwu[1], John Blessing Maina[2], Nwoke Francisca Chinonye[3]
[1,2]Federal University Wukari, Taraba State, Nigeria
[3]National Bureau of Statistics Ibadan, Oyo State, Nigeria
umaruisaac@gmail.com

Article Info:

Submitted:	Revised:	Accepted:	Published:
Jun 17, 2024	Jul 12, 2024	Jul 14, 2024	Jul 18, 2024

Abstract

Human beings and society are susceptible to criminality and so criminality can be likened to an oil that is capable of staining a very white garment when it comes in contact with the white garment. That is why no society can claim to be completely free from crime. The types of criminal behavior depend on the social and economic development of a given society. It is therefore not unexpected that a society at a low level of development tends to experience an upsurge in the rate of violent crimes such as armed robbery, politically motivated killings, the use of illegal weapons, ethnic and religious clashes, and the like. To examine the relationship between the crime rate and the number of prison inmates in Nigeria. Secondary data was used in this study. The data were on various crimes committed within the period of one year and the data were collected from the National Bureau of Statistics. Multiple linear regression modelswere used to carry out the analysis. The result it shows that the top five states with the highest number of criminals and prison populations are Lagos, Rivers, Delta, Abuja, and Kano States.

Keywords: Crime, Prison, Population, Corruption, Violent, Regression models

INTRODUCTION

Crime does not have a universal definition. This is a result of changes in social, political, psychological, and economic conditions. Crime is a public wrong. It is an act of offense that violates the law of the state and is strongly disapproved by society (Sowmyya,2015). Crime is defined as acts or omissions forbidden by law that can be punished by imprisonment or fine.Most Nigerians agree that crime rates and insecurity in the country are very high and that over the last years, the country has experienced a steep rise in crime. Organized and non-organized crime and attendant vices such as smuggling of contrabands, especially firearms, counterfeiting, money-laundering, armed robberies, kidnapping, car hijacking, and human trafficking have become sources of worry for the Nigerian government. Sowmyya,2015stated that no individual is a born criminal, it is the situations and the conditions around the individual that make him act as a criminal. Several causes make an individual turn into a criminal. Likewise, incidents of high-profile crime and politically motivated killings and kidnapping have lately compounded the complexity of the crime situation in the country. An act may be a crime in one society, but not in another (Danbazau, 2007). For example, prostitution, adultery, and homosexuality between consenting adults have been wholly or partially removed from the criminal law in the USA (Feldman, 1997) but are considered crimes in Muslim communities such as Saudi Arabia. The constant changes in time also change the perception of society on crime. The Waverly Encyclopedia defines it as, "An act forbidden by law and for performing which the perpetrator is liable to punishment". The Concise Encyclopedia of Crime and Criminals has defined 'crime' thus: "A crime is an act or default which prejudices the interests of the community and is forbidden by law under pain of punishment.

Over the years the rate of crime in Nigeria has been on the increase and these crimes are being carried out with more perfection and sophistication. This has led to the formation of various vigilante groups, to combat crimes in some parts of the country (Fajemirokun et al., 2006).

Ashby and Ramos (2013), show that crime has a deterrent effect on Foreign Direct Investment (FDI) using a panel of FDI flows into 32 Mexican states coming from 116 countries and police-reported homicide rates. Related evidence has also shown that the overall costs of violent crimes to the economy are higher than the cost of conflicts and civil war (Hoeffler and Fearon, 2014).

Rosenthal and Ross (2010) show that increases in violent crimes during prime times will decrease number of restaurants in the area. This approach is unlikely to be valid given the fact that past crime rates are likely to be endogenous also. Our approach differs from this by using rainfall shocks We argue that crime imposes a cost on firms and that this in turn might inhibit entry of less productive small firms by negatively affecting their expected profits.

Sanni et al (2010) argued that a community with a high rate of criminal and delinquency activities is unattractive or less attractive to both local and foreign investment.

Adetula and Fatunsi (2010) have shown that contact with the prison institution in Nigeria makes the less hardened individual to be more hardened in criminal activities upon release from position.

METHODS

The method used in order to meet the specific aim and objectives of the work is general multiple linear regression model, the dependent variable is the population of person in prison and the independent variable is the four categories of crime. The multiple linear regression models are given as:

$$Y_i = \beta_0 + \beta_i x_1 + \beta_2 x_2 + ... + \beta_k x_k + \ell \qquad (1)$$

Where

$\gamma_{i=}$Population of crime in prison

X_i =Crime against person

X_2 =crime against property

X_3 =crime against lawful authority

X_4 =crime against local acts

X_i Is (nxp) matrix of independent variable?

β Is (2x1) vector of regression coefficient?

ε Is (px1) vector of statistical error?

From $Y = \times \beta + \varepsilon$

Where Y =is an (nx1) vector of the independent variable X is the (nxp), where p=k+1, matrix of independent variables,β is p x1 vector of regression coefficient and ε the vector disturbance.

Assumptions of the model

The following assumptions underlined the multiple regression models:

Before studying the statistical properties of the OLS estimators in the multiple linear regression models, we need to formulate a set of statistical assumptions. Specifically, the set of assumptions that we will formulate are called classical linear model (CLM) assumptions.it is important to note that CLM assumptions are simple, and that the OLS estimators have, under these assumptions, very good properties.

1.Assumption on the functional form

The relationship between the regress and, the regressor and the disturbance in linear in the parameter: $Y_I = \beta_1 + \beta_2 X_2 + ... + \beta_k X_k + u$

Or alternatively, for all the observation, $y = X\beta + u$

2. Assumptions on the regressor:

The values of $X_1, X_2, ..., X_k$ are fixed in repeated sampling or the matrix X is fixed in repeated in sampling.

3. Assumptions of the parameters:

The parameters $\beta_1, \beta_2, \beta_3, ..., \beta_k$ are constant or β is a constant vector.

4. Assumption of disturbances:

$$E(u_I) = 0, i = 1,2,3,...., n$$

The disturbance has zero mean, *or*

$$E(u) = 0$$

The disturbances have a constant variance (homoscedasticity assumption):

$$\mathrm{var}(u_i) = \sigma^2, i = 1,2,3,......, n$$

The disturbances with difference subscript are not correlated with each other (no autocorrelation assumptions).

5. the disturbance term is normally distributed.

6. the number of observations must be greater than the number of repressors.

Model derivation

Using MLE

The basic idea of regression is to estimate the population parameters $\beta_1, \beta_2, \beta_3 ...... \beta_k$ from a given sample.

The sample regression function (SRF) is the sample counterpart of the population regression function (PRF). Since the SRF is obtained for a given sample; a new sample will generate different estimates.

The SRF, which is expressed is an estimates of PRF, given by

$$\bar{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_2 + \hat{\beta}_3 x_{3i} + ... + \hat{\beta}_k x_k \quad i=1, 2,\ldots,n \tag{2}$$

The above expression allows us to calculate the fitted value of

$\hat{y}_i$ *for each* y_i int *the SRF* $\hat{\beta}_i, \hat{\beta}_2, \hat{\beta}_3 .... \beta_k$ *are estimation of the parameters of* $\beta_1 \beta_2 \beta_3 .... \beta_k$

We call residual to the difference between

$$y_i \text{ and } \hat{y}_i \text{ i.e } \hat{u}_i = \hat{y}_i - \hat{y}_i = y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{2i} - \hat{\beta}_3 x_{3i} - .... - \hat{\beta}_k x_{ki} \tag{3}$$

In other words, the residual $\hat{u}_i$ is the difference between a sample value and its corresponding fitted value. The system of equation can be expressed in a compact form using matrix notation. Thus, we are going to denote

$$\hat{y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \cdots \\ \hat{y}_n \end{bmatrix}, \hat{\beta} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_k \end{bmatrix}, \hat{u} = \begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \cdots \\ \hat{u}_n \end{bmatrix}$$

(4)

For the observation of sample, the corresponding fitted model will be the following

$$\hat{y} = X\hat{\beta}$$

(5)

The residual vector is equal to the difference between the vector of observed values and the vector fitted values, that is to say, $\hat{u} = y - \hat{y} - X\hat{\beta}$

(6)

Model Set up

Observation number	Response Y	Explanatory Variables $X_1 X_2 \cdots X_K$
1 **2** **:** **N**	y_1 y_2 : : y_n	$x_{11} x_{12} \cdots x_{1k}$ $x_{21} x_{22} \cdots x_{2k}$: : $x_{n1} x_{n2} \cdots x_{nk}$

Assuming that the model is

$$Y = \beta_0 + \beta_i x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \ell$$

Obtaining the OLS estimates, interpretation of the coefficients, and other characteristics

Obtaining the OLS estimator

Denoting S to the sum of the squared residual,

$$S = \sum_{i=1}^{n} \hat{u}^2{}_i = \sum_{i=1}^{n} \left[y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{2i} - \hat{\beta}_3 x_{3i} - \cdots - \hat{\beta}_k x_{ki} \right]^2$$

(7)

To apply the least square criteria in the model of multiple linear regressions, we calculate the first derivative from S with respect to *B*, in the expression

$$\frac{\delta S}{\delta \hat{\beta}_1} = 2\sum_{i=1}^{n}\left[y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{2i} - \hat{\beta}_3 x_{3i} \cdots\cdots \hat{\beta}_k x_{ki}\right][-1]$$

$$\frac{\delta S}{\delta \hat{\beta}_2} = 2\sum_{I=1}^{n}\left[y_I - \hat{\beta}_1 - \hat{\beta}_2 x_{2i} - \hat{\beta}_3 x_{3i} \cdots\cdots \hat{\beta}_k x_{ki}\right][-x_{2i}]$$

$$\frac{\delta S}{\delta \hat{\beta}_3} = 2\sum_{i=1}^{n}\left[y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{2i} - \hat{\beta}_3 x_{3i} \cdots\cdots \hat{\beta}_k x_{ki}\right][-x_{3i}]$$

....

$$\frac{\delta S}{\delta \hat{\beta}_k} = 2\sum_{i=1}^{n}\left[y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{2i} - \hat{\beta}_3 x_{3i} - \cdots - \hat{\beta}_k x_{ki}\right][-x_{ki}]$$

(8)

The least square estimator is obtained equaling to 0 the previous derivatives

$$\sum_{i=1}^{n}\left[y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{2i} - \hat{\beta}_3 x_{3i} - \cdots - \hat{\beta}_k x_{ki}\right] = 0$$

$$\sum_{i=1}^{n}\left[y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{2i} - \hat{\beta}_3 x_{3i} - \cdots - \hat{\beta}_k x_{ki}\right] x_{2i} = 0$$

$$\sum_{i=1}^{n}\left[y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{2i} - \hat{\beta}_3 x_{3i} - \cdots - \hat{\beta}_k x_{ki}\right] x_{3i} = 0$$

....

$$\sum_{i=1}^{n}\left[y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{2i} - \hat{\beta}_3 x_{3i} - \cdots - \hat{\beta}_k x_{ki}\right] x_{ki} = 0$$

Or in matrix notation

$$X' X\hat{\beta} = X' y$$

(9)

The previous equations are denominated generically hyper plane normal equations.

In expanded matrixes, the system of normal equations is the following:

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_{2i} & \cdots & \sum_{i=1}^{n} x_{ki} \\ \sum_{i=1}^{n} x_{2i} & \sum_{i=1}^{n} x^2{}_{2i} & \cdots & \sum_{i=1}^{n} x_{2i} x_{ki} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{n} x_{ki} & \sum_{i=1}^{n} x_{ki} x_{2i} & \cdots & \sum_{i=1}^{n} x^2{}_{ki} \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{2i} y_i \\ \vdots \\ \sum_{i=1}^{n} x_{ki} y_i \end{bmatrix}$$

(10)

Note that:

a.) X'X/n is the matrix of second order sample moments with respect to the origin, of the regressors, among which the dummy regressor (x1i) associated to the intercept, is included. This regressor takes the value x1i=1 for all i.

b.)X'y/n is the vector of sample moment of second order, with respect to the origin, between the regressand and the regressors. In this system there are k equation and k unknown , $\left(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3 \ldots\ldots, \hat{\beta}_k\right)$

This system can easily be solved using matrix algebra.in order to solve univocally the system with respect to $\hat{\beta}$, it must be held that the rank of the matrix X'X is equal to *k*. if this holds, both members can be premultiplied by $[X'X]^{-1}$:

$$[X'X]^{-1} X'X\hat{\beta} = [X'X]^{-1} X'y$$

With which the expression of the vector of least square estimators, or more precisely, the vector of *ordinary* least square estimators (OLS), is obtained because $[X'X]^{-1} X'X = 1$ Therefore, the solution is the following:

$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_k \end{bmatrix} = \hat{\beta} = [X'X]^{-1} X'y$$

Since the matrix of second derivatives, 2X'X, is a positive definite matrix, the conclusion is that S present a minimum in $\hat{\beta}$.

Interpretation of the coefficients

$\hat{\beta}_j$ Coefficients measures the partial effect of the regressor *Xj* on *y* holding the other regressors fixed. We will see next the meaning of this expression.

The fitted model for observation *i*is given by

$$y_i = \hat{\beta}_1 + \hat{\beta}_2 x_{2i} + \hat{\beta}_3 x_{3i} + \dots\dots\dots + \hat{\beta}_j x_{ji} + \dots\dots\dots + \hat{\beta}_k x_{ki} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

(11)

Now let us consider the fitted model observation *h* in which the values of the regressors and consequently, *y* will have change with respect to

$$\hat{y}_h = \hat{\beta}_1 + \hat{\beta}_2 x_{2h} + \hat{\beta}_3 x_{3h} + \dots\dots\dots + \hat{\beta}_j x_{jh} + \dots\dots\dots + \hat{\beta}_k x_{kj} \dots\dots\dots\dots\dots\dots$$

$$subtracting(2) from(2) wehve$$

$$\Delta y = \hat{\beta}_2 \Delta x_2 + \hat{\beta}_3 \Delta x_3 + \dots\dots\dots + \hat{\beta}_j \Delta x_j + \dots\dots\dots + \hat{\beta}_k \Delta x_k$$

$$where$$

$$\Delta \hat{Y} = \hat{y}_i - \hat{y}_h, \Delta x_2 = x_{2i} - x_{2h}, \Delta x_3 = x_{3i} - x_{3h}, \dots\dots\dots\dots, \Delta x_k = x_{ki} - x_{kh}.$$

(12)

The previous expression captures the variation of *y* due to the change in all regressors. If only *Xj*changes, we will have $\Delta \hat{y} = \hat{\beta}_j \Delta x_j \, if \, x_j$ Increase in one unite we have

$$\Delta \hat{y} = \hat{\beta}_j \; for \Delta x_j = 1$$

RESULTS AND DISCUSSION

The analysis of the data and the results obtained are presented. The multiple regression models analysis was carried out using R software with the following packages installed from R Studio: ggplot2, reshape2, and lm. These packages enabled us to visualize the data using plots.

Data visualization

The data was visualized using the melt () and ggplot() packages in R. As shown in the figure below:

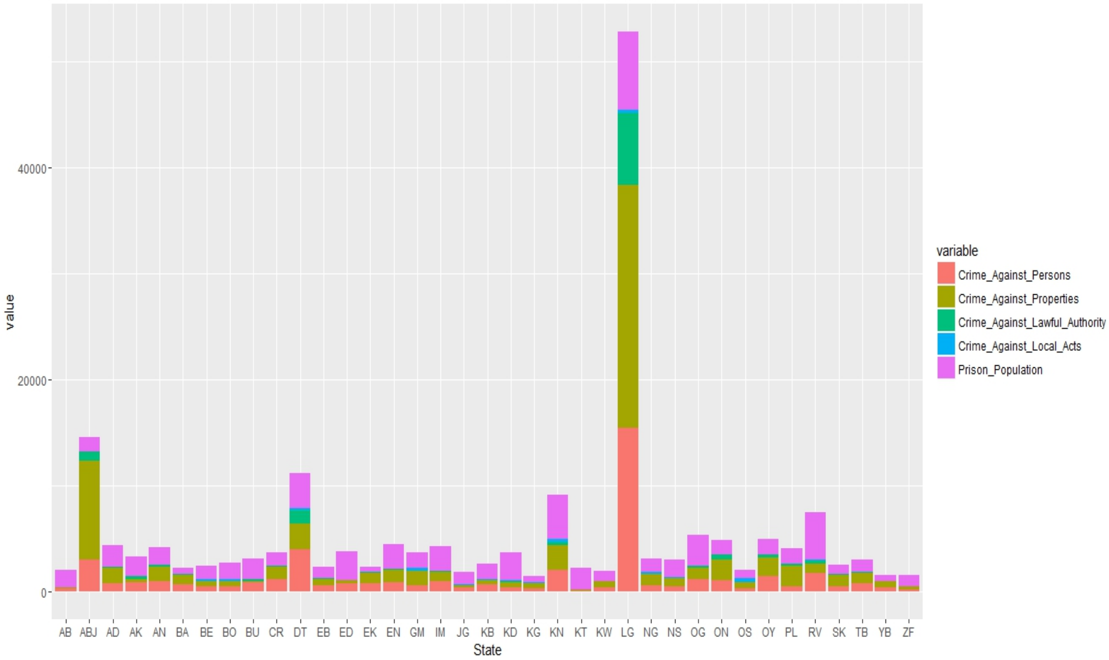

Figure 1

From the figure above we observed that the top five states with the highest number of criminals and prison populations are Lagos, Rivers, Delta, Abuja and Kano States.

Table 1

Variables / Model	Coefficient	SE	T	P-value
Constant	943.5666	228.7027	4.126	0.000246
Crime against person	1.5328	0.3959	3.871	0.000502
Crime against property	-0.2724	0.1133	-2.403	0.022215
Crime against lawful authority	-1.6540	0.8544	-1.936	0.061758
Crime against local acts	0.4258	1.2691	0.336	0.739403
Residual std error 729.7 R-Squared=0.7249	F-Statistics=21.08 adjusted R-Squared=0.6905		P-Value=1.354e-08	

From the table above we observed that, every unite change in the number of crime committed leads to increase by 1.5328 of crime against person X_1, -0.2724 of crime against property X_2, -1.6540 of crime against lawful authority X_3 and 0.4258 crime against local acts X_4.with the constant :

$$Y_i = 943.566 + 1.5328\,X_1 + 0.2724\,X_2 + (-1.6540)\,X_3 + 0.4258\,X_4$$

Where X_1=Crime against person, X_2=Crime against property, X_3=Crime against lawful authority and X_4=Crime against local acts.

Interpretation

943.566 is the intercept on constant term where the regression line cuts the Y-axis which is the number of prison inmates at X =0.Also, every unite change in the number of crime committed leads to increase by 1.5328 of crime against person X_1, -0.2724 of crime against property X_2, -1.6540 of crime against lawful authority X_3 and 0.4258 crime against local acts X_4.

Stating the hypothesis

$$H0: \beta_i = 0$$
$$H1: \beta_i \neq 0$$

Alternatively

$$Ho: \beta_0 = \beta_I = 0$$
$$Hi: \beta_0 \neq \beta_{i_i} \neq 0$$

1. Ho: There is a significant difference in the rate of crime committed in the various states

2. H1: There is no significant difference in the crime committed in the states

Decision Rule

If T-cal>T-tab at 95% level of significance (i.e. $\alpha = 0.05$) We reject Ho and accept H1 and conclude that there are no significant difference in the rate of crime committed against person, property, lawful authority and local acts.

Since P-value (0.0000502) $< \alpha -$value (0.5), the P $< \alpha$ Coefficient of crime against person contributed significantly to prison inmates, P-value (0.022215) $< \alpha -$value (0.5), the P$< \alpha$ coefficient of crime againstproperty contributed significantly to the prison inmates,

P-value (0.061758)$< \alpha -$value (0.5), the P$< \alpha$ coefficient of crime againstlawful authority contributed significantly to the prison inmates, P-value (0.739403)$> \alpha -$value (0.5), the P> α coefficient of crime against local acts which contributed insignificantly to the number of prison inmates in the country.

CONCLUSION

Poverty remains a key challenge and the main cause of rise in crime in most of the development societies. Poverty results out of bad leadership. In the Nigeria case, it is failure of leadership both at national and state levels. Ours is a case of resource cause theory.

From petty crime to high-scale security threats, the situation appears to be getting out of hand. Armed groups are having a field day so also are other criminal elements devising new means of remaining in the trade. If the situation is not quickly curtailed, it will harshly hamper the growth of Nigeria's economy, thereby showing the pace of development across the densely populated nation. Nobody wants to invest or do business in a place that is not safe. The poor state of security is threatening the growth of the economy. The government should go beyond periodic donations of patrol vehicles, gunboats and other crime fighting equipment and critically address the core issues bedeviling the force and the society at large which of course is poverty.

Conflict of Interest

There is no conflict of interest

Acknowledgement

We are grateful to the Federal University, Wukari for providing a wonderful and enabling environment for us which makes it possible for this work to be carried out.

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