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A Bioeconomic Prey-Predator Model with a Water-Pollution Modification Parameter and Stability Analysis

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Abstract

Managing interacting fish populations requires consideration of ecological processes and human pressures across reserve and unreserved zones. This study developed and analyzed a bioeconomic predator–prey model for three interacting fish populations: cichlids, tilapia, and Nile perch. The model incorporated water pollution, harvesting, migration, and predation across both zones. Equilibrium points and their existence conditions were derived, and the local stability of selected equilibria was assessed using Jacobian linearization and the Routh–Hurwitz criterion. Numerical simulations examined the effects of pollution control, predation control, and a combined strategy integrating water-quality improvement and migration. The results indicated that stronger pollution control enhanced fish population persistence in the unreserved zone. The combined strategy produced greater improvements than individual interventions, while predator–prey interactions remained integral to the modeled ecosystem. These findings highlight the potential benefits of coordinating water-quality improvement and migration management to support fish population persistence. The model provides an analytical framework for evaluating management strategies that account for pollution, harvesting, and ecological interactions in aquatic systems with reserve and unreserved zones.

Keywords: Bioeconomic Modeling; Predator–Prey Dynamics; Local Stability; Water Pollution; Fishery Management

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INTRODUCTION

Fish inhabit marine, freshwater, and other aquatic environments. They are ectothermic vertebrates characterised by gills, fins, and streamlined bodies adapted for swimming (Badoni *et al.*, 2021; Verma & Prakash, 2020). Fish also constitute an important source of high-quality protein and support employment, income generation, and recreational activities (Umar & Musa, 2017).

Fish contribute substantially to human nutrition and economic activity. Their flesh is generally rich in protein, iodine, vitamins A and D, calcium, and long-chain omega-3 polyunsaturated fatty acids. Consequently, fish provide a vital source of animal protein for more than 3.5 billion people worldwide (Food & Agriculture Organization, 2020).

Beyond direct consumption, fisheries create opportunities for employment and self-employment and generate commercially valuable by-products, including fertiliser, fish glue, and fish oil. These products are used in industries such as painting, soap production, and medicine. The fish and seafood market was projected to generate US$71.50 billion in revenue in 2025 and to grow annually by 6.22% from 2025 to 2029. Global fish trade was valued at approximately US$150 billion in 2020 (FAO, 2022), with China accounting for a substantial share of global market revenue.

Fish are distributed across the Atlantic, Indian, Arctic, and other oceanic and inland-water systems. Global fisheries and aquaculture production reached approximately 179 million metric tonnes in 2020, of which aquaculture contributed more than 114 million metric tonnes (Eurostat, 2023; FAO, 2022). Approximately 34,000 fish species have been identified, and additional species continue to be documented (FishBase, 2020).

Fish populations are declining in many regions because of overfishing, destructive fishing practices, habitat degradation, climate change, invasive species, and water pollution (Cowx *et al.*, 2025; FAO, 2024; Umar & Musa, 2017). Mathematical and management studies have proposed several responses to these pressures. Raymond, Hugo, and Kung'aro (2019) analysed a two-prey, one-predator fishery model with a Holling type II functional response. Zhou *et al.* (2019) argued that ecosystem-based harvesting can support sustainable yields while reducing ecological disruption. Free *et al.* (2022) further showed that harvest control rules can strengthen climate resilience by incorporating uncertainty and ecological variability. Yoshioka (2023) proposed a control framework that accounts for uncertainty and population

heterogeneity, while balanced harvesting distributes fishing pressure proportionally across ecosystem components to preserve ecosystem structure and function.

Umar and Musa (2017) developed a prey-predator model for a polluted environment with reserve zones. However, their model included only two interacting populations and did not explicitly incorporate a strategy for reducing water pollution in the unreserved zone. Because multispecies interactions better reflect ecosystem complexity and water pollution is a major source of fish mortality, the present study extends that framework to three populations and introduces a water-pollution modification parameter. The model is intended to clarify how pollution control, migration, harvesting, and predation jointly influence fish population dynamics.

METHODOLOGY

Model Formulation

The model extends Umar and Musa (2017) and is formulated under the following assumptions:

i. The model consists of three interacting fish populations.
ii. No movement occurs between reserve zones.
iii. Water in the reserve zone is assumed to be free from pollution.
iv. Predation occurs only in the unreserved zone.
v. Natural mortality is lower in the reserve zone than in the unreserved zone.
vi. A common water-pollution modification parameter applies to the populations in the unreserved zone.

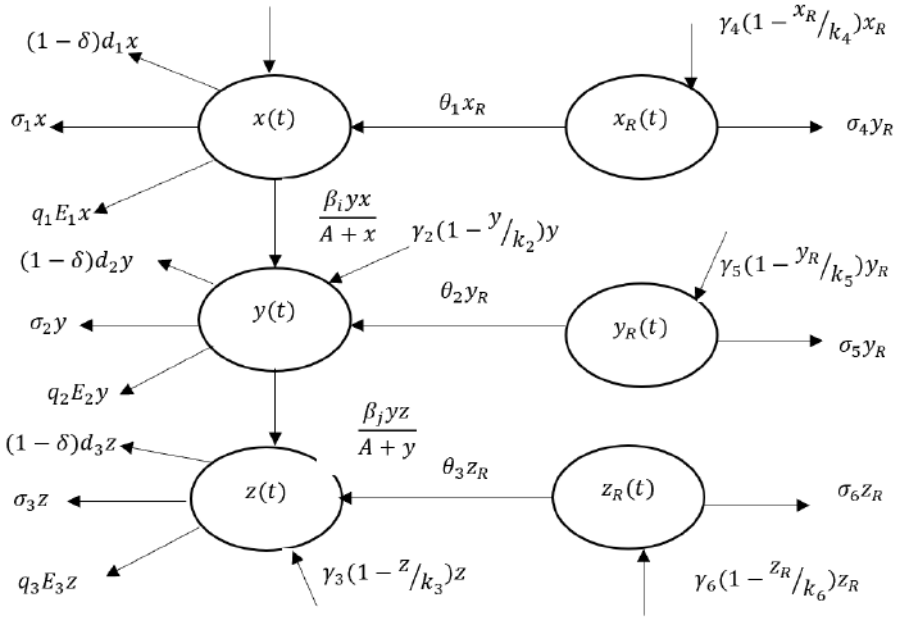
Here, $x(t)$, $y(t)$, $z(t)$, $x_R(t)$, $y_R(t)$, and $z_R(t)$ denote the fish populations at time t in the unreserved and reserve zones, respectively. Specifically, they represent Cichlid, Tilapia, and Nile perch in each zone. The economic-rent variables are $E_C(t)$, $E_T(t)$, and $E_N(t)$. The model parameters are defined in Table 1.

Table 1. Model Parameters

Parameters	Description
$\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6$	Intrinsic growth rates of Cichlid, Tilapia, and Nile perch in the unreserved and reserve zones
$k_1, k_2, k_3, k_4, k_5, k_6$	Environmental carrying capacities for Cichlid, Tilapia, and Nile perch in the unreserved and reserve zones

$\boldsymbol{\beta_1}$	Maximum predation effect of y on x
$\boldsymbol{\beta_2}$	Maximum relative predation benefit of y from consuming x
$\boldsymbol{\beta_3}$	Maximum predation effect of z on y
$\boldsymbol{\beta_4}$	Maximum relative predation benefit of z from consuming y
$\boldsymbol{d_1, d_2, d_3}$	Water-pollution-induced mortality rates of Cichlid, Tilapia, and Nile perch
$\boldsymbol{A}$	Saturation constant
$\boldsymbol{q_1, q_2, q_3}$	Catchability coefficients of Cichlid, Tilapia, and Nile perch
$\boldsymbol{\mu_1, \mu_2, \mu_3}$	Stiffness parameters for Cichlid, Tilapia, and Nile perch
$\mathbf{p_1, p_2, p_3}$	Constant prices per unit biomass for Cichlid, Tilapia, and Nile perch
$\boldsymbol{c_1, c_2, c_3}$	Constant costs per unit biomass for Cichlid, Tilapia, and Nile perch
$\boldsymbol{\delta}$	Water-pollution modification parameter
$\boldsymbol{\theta_1, \theta_2, \theta_3}$	Migration rates of Cichlid, Tilapia, and Nile perch from the reserve zone to the unreserved zone
$\boldsymbol{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6}$	Natural mortality rates of Cichlid, Tilapia, and Nile perch in the unreserved and reserve zones
$\boldsymbol{E_1, E_2, E_3}$	Harvesting efforts for Cichlid, Tilapia, and Nile perch

Schematic Diagram of the Model



Model Equations

$$\dot{x}(t)=\gamma_1\left(1-\frac{x}{k_1}\right)x-q_1E_1x-\frac{\beta_1yx}{A+x}-(1-\delta)d_1x-\sigma_1x+\theta_1x_R \tag{1}$$

$$\dot{y}(t)=\gamma_2\left(1-\frac{y}{k_2}\right)y-q_2E_2y+\frac{\beta_2yx}{A+x}-\frac{\beta_3zy}{A+y}-(1-\delta)d_2y-\sigma_2y+\theta_2y_R \tag{2}$$

$$\dot{z}(t)=\gamma_3\left(1-\frac{z}{k_3}\right)z-q_3E_3z+\frac{\beta_4zy}{A+y}-(1-\delta)d_3z-\sigma_3y+\theta_3z_R \tag{3}$$

$$\dot{x}_R(t)=\gamma_4\left(1-\frac{x_R}{k_4}\right)x_R-\sigma_4x_R-\theta_1x_R \tag{4}$$

$$\dot{y}_R(t)=\gamma_5\left(1-\frac{y_R}{k_5}\right)y_R-\sigma_5y_R-\theta_2y_R \tag{5}$$

$$\dot{z}(t)=\gamma_6\left(1-\frac{z_R}{k_6}\right)z_R-\sigma_6z_R-\theta_3z_R \tag{6}$$

$$\dot{E}_C(t)=\mu_1(p_1q_1x-c_1)E_1 \tag{7}$$

$$\dot{E}_T(t)=\mu_2(p_2q_2y-c_2)E_2 \tag{8}$$

$$\dot{E}_N(t)=\mu_3(p_3q_3z-c_3)E_3 \tag{9}$$

Subject to the initial conditions

$x(0)\geq,\ y(0)\geq,\ z(0)\geq,\ x_R(0)\geq,\ y_R(0)\geq,\ z_R(0)\geq, E_C(0)\geq,\ E_T(0)\geq,\ E_N(0)$.

Model Analysis

Using the notational definitions introduced below, equations (1)-(9) can be rewritten as follows:

$$\dot{x}(t)=\omega_1x-\frac{\gamma_1}{k_1}x^2-q_1E_1x-\frac{\beta_1yx}{A+x}+\theta_1x_R \tag{10}$$

$$\dot{y}(t)=\omega_2y-\frac{\gamma_2}{k_2}y^2-q_2E_2y+\frac{\beta_2yx}{A+x}-\frac{\beta_3zy}{A+y}+\theta_2y_R \tag{11}$$

$$\dot{z}(t) = \omega_3 z - \frac{\gamma_3}{k_3} z^2 - q_3 E_3 z + \frac{\beta_4 zy}{A+y} + \theta_3 z_R \tag{12}$$

$$\dot{x}_R(t) = \omega_4 x_R - \frac{\gamma_4}{k_4} x_R^2 - \theta_1 x_R \tag{13}$$

$$\dot{y}_R(t) = \omega_5 y_R - \frac{\gamma_5}{k_5} y_R^2 - \theta_2 y_R \tag{14}$$

$$\dot{z}(t) = \omega_6 z_R - \frac{\gamma_6}{k_6} z_R^2 - \theta_3 z_R \tag{15}$$

$$\dot{E}_C(t) = \mu_1 (p_1 q_1 x - c_1) E_1 \tag{16}$$

$$\dot{E}_T(t) = \mu_2 (p_2 q_2 y - c_2) E_2 \tag{17}$$

$$\dot{E}_N(t) = \mu_3 (p_3 q_3 z - c_3) E_3 \tag{18}$$

For notational convenience, define:

$\omega_1 = \left(\gamma_1 - (1-\delta)d_1 - \sigma_1\right)$, $\omega_2 = \left(\gamma_2 - (1-\delta)d_2 - \sigma_2\right)$, $\omega_3 = \left(\gamma_3 - (1-\delta)d_3 - \sigma_3\right)$, $\omega_4 = \left(\gamma_4 - \theta_1 - \sigma_4\right)$, $\omega_5 = \left(\gamma_5 - \theta_2 - \sigma_5\right)$, and $\omega_6 = \left(\gamma_6 - \theta_3 - \sigma_6\right)$

Equilibrium Points and Their Existence Conditions

This section derives the equilibrium points of equations (10)-(18) and establishes the conditions required for their existence. Solving the system yields thirteen equilibrium points, listed below:

$$P_1\left(\xi_1, 0, 0, 0, 0, 0, \left(\frac{\omega_1 p_1}{c_1} - \frac{\gamma_1}{k_1 q_1}\right), 0, 0\right),\ P_2\left(0, \xi_2, 0, 0, 0, 0, 0, \left(\frac{\omega_2 p_2}{c_2} - \frac{\gamma_2}{k_2 q_2}\right), 0\right),$$

$$P_3\left(0, 0, \xi_3, 0, 0, 0, 0, 0, \left(\frac{\omega_3 p_3}{c_3} - \frac{\gamma_3}{k_3 q_3}\right)\right),\ P_4\left(\xi_1, 0, 0, \xi_4, 0, 0, \frac{1}{q_1}\left(\omega_1 - \frac{\gamma_4}{k_4}\xi_1 + \theta_1\right), 0, 0\right),$$

$$P_5\left(0, \xi_2, 0, 0, \xi_5, 0, 0, \frac{1}{q_2}\left(\omega_2 - \frac{\gamma_5}{k_5}\xi_2 + \theta_2\right), 0, 0\right),\ P_6\left(0, 0, \xi_3, 0, 0, \xi_6, 0, 0, \frac{1}{q_3}\left(\omega_3 - \frac{\gamma_6}{k_6}\xi_3 + \theta_3\right)\right)$$

$$,\ P_7\left(\xi_1, \xi_2, 0, 0, 0, 0, \frac{1}{q_1}\left(\omega_1 - \frac{\gamma_1}{k_1}\xi_1 + \frac{\beta_1 \xi_2}{A+\xi_1}\right), \frac{1}{q_2}\left(\omega_2 - \frac{\gamma_2}{k_2}\xi_2 + \frac{\beta_2 \xi_1}{A+\xi_2}\right), 0\right),$$

$$P_8\left(0,\xi_2,\xi_3,0,0,0,0,\frac{1}{q_2}\left(\omega_2-\frac{\gamma_2}{k_2}\xi_2+\frac{\beta_3\xi_3}{A+\xi_2}\right),\frac{1}{q_3}\left(\omega_3-\frac{\gamma_3}{k_3}\xi_3+\frac{\beta_4\xi_2}{A+\xi_2}\right)\right),$$

$$P_9\left(\xi_1,0,\xi_3,0,0,0,\frac{1}{q_1}\left(\omega_1-\frac{\gamma_1}{k_1}\xi_1\right),0,\frac{1}{q_3}\left(\omega_3-\frac{\gamma_3}{k_3}\xi_3\right)\right),$$

$$P_{10}\left(\xi_1,\xi_2,0,\xi_4,\xi_5,0,\frac{1}{q_1}\left(\omega_1-\frac{\gamma_1}{k_1}\xi_1-\frac{\beta_1\xi_2}{A+\xi_1}+\theta_1\xi_4\right),\frac{1}{q_2}\left(\omega_2-\frac{\gamma_2}{k_2}\xi_2+\frac{\beta_2\xi_1}{A+\xi_1}+\theta_2\xi_5\right),0\right),$$

$$P_{11}\left(0,\xi_2,\xi_3,0,\xi_5,\xi_6,0,\frac{1}{q_2}\left(\omega_2-\frac{\gamma_2}{k_2}\xi_2-\frac{\beta_3\xi_3}{A+\xi_2}+\theta_2\xi_5\right),\frac{1}{q_3}\left(\omega_3-\frac{\gamma_3}{k_3}\xi_3+\frac{\beta_4\xi_2}{A+\xi_2}+\theta_3\xi_6\right)\right),$$

$$P_{12}\left(\xi_1,0,\xi_3,\xi_4,0,\xi_6,\frac{1}{q_1}\left(\omega_1-\frac{\gamma_1}{k_1}\xi_1+\theta_1\xi_4\right),0,\frac{1}{q_3}\left(\omega_3-\frac{\gamma_3}{k_3}\xi_3+\theta_3\xi_6\right)\right),\text{ and}$$

$$P^*=P_{13}\left(\xi_1,\xi_2,\xi_3,\xi_4,\xi_5,\xi_6,\xi_7,\xi_8,\xi_9\right)$$

where $\xi_1=\frac{c_1}{p_1q_1},\xi_2=\frac{c_2}{p_2q_2},\xi_3=\frac{c_3}{p_3q_3},\xi_4=\frac{k_4}{\gamma_4}\left(\omega_4-\theta_1\right),\xi_5=\frac{k_5}{\gamma_5}\left(\omega_5-\theta_2\right),\ \xi_6=\frac{k_6}{\gamma_6}\left(\omega_6-\theta_3\right),$

$$\xi_7=\frac{1}{q_1}\left(\omega_1-\frac{\gamma_1}{k_1}\xi_1-\frac{\beta_1\xi_2\xi_1}{A+\xi_1}+\theta_1\xi_4\right),\ \xi_8=\frac{1}{q_2}\left(\omega_2-\frac{\gamma_2}{k_2}\xi_2+\frac{\beta_2\xi_2\xi_1}{A+\xi_1}-\frac{\beta_3\xi_2\xi_1}{A+\xi_2}+\theta_2\xi_5\right),\text{ and}$$

$$\xi_9=\frac{1}{q_3}\left(\omega_3-\frac{\gamma_3}{k_3}\xi_3+\frac{\beta_4\xi_3\xi_2}{A+\xi_2}+\theta_3\xi_6\right)$$

Hence, $P^*=P_{13}\left(x^*,y^*,z^*,x_R^*,y_R^*,z_R^*,E_C^*,E_T^*,E_N^*\right)$ is the coexistence equilibrium point.

Existence Conditions for the Equilibrium Points

The equilibrium point $P_0\left(0,0,0,0,0,0\right)$ is trivial and represents the extinction of all fish populations. The equilibrium points P_1,P_2,P_3,P_4,P_5 and P_6 exist if and only if $\omega_1,\ \omega_2,\ \omega_3,\ \omega_4>0$; equivalently, the inequalities in (19) must hold.

$$\left.\begin{array}{l}\left(\gamma_1-(1-\delta)d_1-\sigma_1\right)>0,\ \left(\gamma_2-(1-\delta)d_2-\sigma_2\right)>0\\ \left(\gamma_{13}-(1-\delta)d_3-\sigma_3\right)>0,\ \left(\gamma_4-(1-\delta)d_4-\sigma_4\right)>0\end{array}\right\}\qquad(19)$$

The additional inequalities in (20) provide the corresponding positivity conditions.

$$\left.\begin{aligned} &\frac{\omega_1 p_1}{c_1} > \frac{\gamma_1}{k_1 q_1},\ \frac{\omega_2 p_2}{c_2} > \frac{\gamma_2}{k_2 q_2},\ \frac{\omega_3 p_3}{c_3} > \frac{\gamma_3}{k_3 q_3} \\ &\left(\omega_1 + \theta_1\right) > \frac{\gamma_4}{k_4}\xi_1,\ \left(\omega_2 + \theta_2\right) > \frac{\gamma_5}{k_5}\xi_2,\ \left(\omega_3 + \theta_3\right) > \frac{\gamma_6}{k_6}\xi_3 \end{aligned}\right\} \quad (20)$$

The equilibrium points P_7 and P_8 must also satisfy (21).

$$\left.\left(\omega_1 + \frac{\beta_1\xi_2}{A+\xi_1}\right) > \frac{\gamma_1}{k_1}\xi_1,\ \left(\omega_2 + \frac{\beta_2\xi_1}{A+\xi_2}\right) > \frac{\gamma_2}{k_2}\xi_2,\ \left(\omega_3 + \frac{\beta_4\xi_2}{A+\xi_2}\right) > \frac{\gamma_3}{k_3}\xi_3 \right\} \quad (21)$$

Similarly, the equilibrium point P_9 must satisfy (22).

$$\left.\omega_1 > \frac{\gamma_1}{k_1}\xi_1,\ \omega_3 > \frac{\gamma_3}{k_3}\xi_3 \right\} \quad (22)$$

The equilibrium points $P_{10}, P_{11},$ and P_{12} must satisfy (23).

$$\left.\begin{aligned} &\left(\omega_1 + \theta_1\xi_4\right) > \left(\frac{\gamma_1}{k_1}\xi_1 + \frac{\beta_1\xi_2}{A+\xi_1}\right),\ \left(\omega_2 + \theta_2\xi_5\right) > \left(\frac{\gamma_2}{k_2}\xi_2 + \frac{\beta_3\xi_3}{A+\xi_2}\right) \\ &\left(\omega_2 + \theta_2\xi_5 + \frac{\beta_2\xi_1}{A+\xi_1}\right) > \frac{\gamma_2}{k_2}\xi_2,\ \left(\omega_3 + \theta_3\xi_6 + \frac{\beta_4\xi_2}{A+\xi_2}\right) > \frac{\gamma_3}{k_3}\xi_3 \\ &\left(\omega_1 + \theta_1\xi_4\right) > \frac{\gamma_1}{k_1}\xi_1,\ \left(\omega_3 + \theta_3\xi_6\right) > \frac{\gamma_3}{k_3}\xi_3 \end{aligned}\right\} \quad (23)$$

Proposition 1

The equilibrium points P_1, P_2, P_3, P_4, P_5 and P_6 satisfy the conditions in inequalities (19) and (20). In addition, P_7 and P_8 satisfy inequality (21). Finally, P_9, P_{10}, P_{11} and P_{12} satisfy the conditions in inequalities (22) and (23).

Local Stability Analysis of Selected Equilibrium Points

Local stability is examined by linearizing the system and applying the Routh-Hurwitz criterion. The Jacobian matrix of equations (10)-(18) is evaluated at some selected equilibrium point. An equilibrium is locally asymptotically stable when every eigenvalue of the evaluated Jacobian has a negative real part; it is unstable when at least one eigenvalue has a positive real part. The analysis proceeds as follows.

Theorem 1

An equilibrium point x^* of a differential-equation system is locally asymptotically stable if every eigenvalue of the Jacobian J^* evaluated at x^* has a negative real part. The equilibrium is unstable if at least one eigenvalue has a positive real part.

Proof

The Jacobian matrix corresponding to equations (10)-(18) is given in (24).

$$J=\begin{bmatrix}
\omega_1-\frac{2\gamma_1}{k_1}x-q_1E_1+\frac{A\beta_1 y}{(A+x)^2} & -\frac{\beta_1 x}{(A+x)} & 0 & \theta_1 & 0 & 0 & -q_1x & 0 & 0\\
-\frac{A\beta_2 y}{(A+x)^2} & \omega_2-\frac{2\gamma_2}{k_2}y-q_2E_2+\frac{\beta_2 x}{(A+x)}-\frac{A\beta_2 z}{(A+y)^2} & -\frac{\beta_3 y}{(A+y)} & 0 & \theta_2 & 0 & 0 & -q_2y & 0\\
0 & \frac{A\beta_4 z}{(A+y)^2} & \omega_3-\frac{2\gamma_3}{k_3}z-q_3E_3+\frac{\beta_4 y}{(A+y)} & 0 & 0 & \theta_3 & 0 & 0 & -q_2z\\
0 & 0 & 0 & \omega_4-\frac{2\gamma_4}{k_4}x_R-\theta_1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \omega_5-\frac{2\gamma_5}{k_5}y_R-\theta_2 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \omega_6-\frac{2\gamma_6}{k_6}z_R-\theta_3 & 0 & 0 & 0\\
\mu_1p_1q_1E_1 & 0 & 0 & 0 & 0 & 0 & \mu_1(p_1q_1x-c_1) & 0 & 0\\
0 & \mu_2p_2q_2E_2 & 0 & 0 & 0 & 0 & 0 & \mu_2(p_2q_2y-c_2) & 0\\
0 & 0 & \mu_3p_3q_3E_3 & 0 & 0 & 0 & 0 & 0 & \mu_3(p_3q_3z-c_3)
\end{bmatrix} \quad (24)$$

Local Stability of the Equilibrium Point P_1

Evaluating (24) at P_1 gives the Jacobian matrix in (25).

$$J(P_1)=\begin{bmatrix}
\omega_1-\frac{2\gamma_1}{k_1}\xi_1-q_1\left(\frac{\omega_1p_1}{c_1}-\frac{\gamma_1}{k_1q_1}\right) & -\frac{\beta_1\xi_1}{(A+\xi_1)} & 0 & \theta_1 & 0 & 0 & -q_1\xi_1 & 0 & 0\\
0 & \omega_2+\frac{\beta_2\xi_1}{(A+\xi_1)} & 0 & 0 & \theta_2 & 0 & 0 & 0 & 0\\
0 & 0 & \omega_3 & 0 & 0 & \theta_3 & 0 & 0 & 0\\
0 & 0 & 0 & \omega_4-\theta_1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \omega_5-\theta_2 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \omega_6-\theta_3 & 0 & 0 & 0\\
\mu_1p_1q_1\left(\frac{\omega_1p_1}{c_1}-\frac{\gamma_1}{k_1q_1}\right) & 0 & 0 & 0 & 0 & 0 & \mu_1(p_1q_1\xi_1-c_2) & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_2c_2 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_3c_3
\end{bmatrix} \quad (25)$$

Transforming matrix (25) into upper-triangular form gives (26).

$$J(P_1)=\begin{bmatrix} -\frac{g_1}{k_1c_1q_1} & -\frac{\beta_1\xi_1}{(A+\xi_1)} & 0 & \theta_1 & 0 & 0 & -q_1\xi_1 & 0 & 0 \\ 0 & \frac{A\omega_2+\beta_2\xi_1+\omega_2\xi_1}{A+\xi_1} & 0 & 0 & \theta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_1 & 0 & 0 & \theta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_4-\theta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_5-\theta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_6-\theta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu_1}{g_1}\left(g_2p_1q_1k_1c_1q_1^{\,2}\xi_1-g_3-g_4\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_2c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_3c_3 \end{bmatrix} \quad (26)$$

where:

$$g_1 = 2c_1\gamma_1q_1\xi_1 - c_1k_1\omega_1q_1 + k_1\omega_1p_1q_1 - c_1\gamma_1$$

$$g_2 = \left(-\frac{-k_1\omega_1p_1q_1 + c_1\gamma_1}{k_1c_1q_1}\right)$$

$$g_3 = 2c_1\gamma_1p_1q_1^{\,2}\xi_1^{\,2} + c_1k_1\omega_1p_1q_1^{\,2}\xi_1 - k_1\omega_1p_1^{\,2}q_1^{\,2}\xi_1 + 2c_1^{\,2}\gamma_1q_1\xi_1$$

$$g_4 = c_1^{\,2}k_1\omega_1q_1 + c_1\gamma_1p_1q_1\xi_1 + c_1k_1\omega_1p_1q_1 - c_1^{\,2}\gamma_1$$

The characteristic equation associated with matrix (26) is given in (27).

$$|J(P_1)-\lambda|=\begin{vmatrix} -\frac{g_1}{k_1c_1q_1}-\lambda & -\frac{\beta_1\xi_1}{(A+\xi_1)} & 0 & \theta_1 & 0 & 0 & -q_1\xi_1 & 0 & 0 \\ 0 & \frac{A\omega_2+\beta_2\xi_1+\omega_2\xi_1}{A+\xi_1}-\lambda & 0 & 0 & \theta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_1-\lambda & 0 & 0 & \theta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_4-\theta_1-\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_5-\theta_2-\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_6-\theta_3-\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu_1}{g_1}\left(g_2p_1q_1k_1c_1q_1^{\,2}\xi_1-g_3-g_4\right)-\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_2c_2-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_3c_3-\lambda \end{vmatrix}=0 \quad (27)$$

Because matrix (27) is upper triangular, its eigenvalues are its diagonal entries:

$$\lambda_1 = -\frac{g_1}{k_1c_1q_1},\ \lambda_2 = \frac{A\omega_2+\beta_2\xi_1+\omega_2\xi_1}{A+\xi_1},\ \lambda_3=\omega_1,\ \lambda_4=\omega_4-\theta_1,\ \lambda_5=\omega_5-\theta_2,\ \lambda_6=\omega_6-\theta_3,$$

$$\lambda_7 = -\frac{\mu_1}{g_1}\left(g_2p_1q_1k_1c_1q_1^{\,2}\xi_1-g_3-g_4\right),\ \lambda_8=-\mu_2c_2,\ \text{and}\ \lambda_9=-\mu_3c_3$$

The eigenvalues $\lambda_1, \lambda_7, \lambda_8,$ and λ_9 are strictly negative. The remaining eigenvalues, $\lambda_2, \lambda_3, \lambda_4, \lambda_5,$ and λ_6, are negative provided the conditions in (28) hold.

$$\left.\begin{aligned} &\omega_1=\left(\gamma_1-(1-\delta)d_1-\sigma_1\right)<0,\ \omega_2=\left(\gamma_2-(1-\delta)d_2-\sigma_2\right)<0 \\ &\omega_2\left(A+\xi_1\right)+\beta_2\xi_1<0,\ \omega_4<\theta_1,\ \omega_5<\theta_2,\ \omega_6<\theta_3 \end{aligned}\right\} \quad (28)$$

If conditions (28) hold, the equilibrium point P_1 is locally asymptotically stable. Biologically, this implies that the Cichlid prey population can persist without invasion by the predator populations.

Local Stability of the Coexistence Equilibrium Point P^*

Evaluating (24) at P^* gives the Jacobian matrix in (29):

$$J(P^*)=\begin{bmatrix} m_1 & -\frac{\beta_1\xi_1}{(A+\xi_1)} & 0 & \theta_1 & 0 & 0 & -q_1x & 0 & 0 \\ -\frac{A\beta_1\xi_2}{(A+\xi_1)^2} & m_2 & -\frac{\beta_3\xi_2}{(A+\xi_2)} & 0 & \theta_2 & 0 & 0 & -q_2y & 0 \\ 0 & \frac{A\beta_4\xi_3}{(A+\xi_2)^2} & m_3 & 0 & 0 & \theta_3 & 0 & 0 & -q_2z \\ 0 & 0 & 0 & m_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5= & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 & 0 & 0 & 0 \\ \mu_1p_1q_1\xi_7 & 0 & 0 & 0 & 0 & 0 & \mu_1(p_1q_1\xi_1-c_1) & 0 & 0 \\ 0 & \mu_2p_2q_2\xi_8 & 0 & 0 & 0 & 0 & 0 & \mu_2(p_2q_2\xi_2-c_2) & 0 \\ 0 & 0 & \mu_3p_3q_3\xi_9 & 0 & 0 & 0 & 0 & 0 & \mu_3(p_3q_3\xi_3-c_3) \end{bmatrix} \quad (29)$$

where $m_1=\omega_1-\frac{2\gamma_1}{k_1}\xi_1-q_1\xi_8+\frac{A\beta_1\xi_2}{(A+\xi_1)^2}$, $m_2=\omega_2-\frac{2\gamma_2}{k_2}\xi_2-q_2\xi_8+\frac{\beta_2\xi_1}{(A+\xi_1)}-\frac{A\beta_3\xi_3}{(A+\xi_2)^2}$,

$m_3=\omega_3-\frac{2\gamma_3}{k_3}\xi_3-q_3\xi_9+\frac{\beta_4\xi_2}{(A+\xi_2)}$, $m_4=\omega_4-\frac{2\gamma_4}{k_4}\xi_4-\theta_1$, $m_5=\omega_5-\frac{2\gamma_5}{k_5}\xi_5-\theta_2$,

$m_6=\omega_6-\frac{2\gamma_6}{k_6}\xi_6-\theta_3$, $m_7=\frac{\beta_1\xi_1}{(A+\xi_1)}$, $m_8=\frac{A\beta_1\xi_2}{(A+\xi_1)^2}$, $m_9=\frac{\beta_3\xi_2}{(A+\xi_2)}$, and $m_{10}=\frac{A\beta_4\xi_3}{(A+\xi_2)^2}$

Applying the Routh-Hurwitz criterion to the characteristic polynomial of $J(P^*)$ in (29) yields (30).

$$f(\lambda)=\lambda^9+a_1\lambda^8+a_2\lambda^7+a_3\lambda^6+a_4\lambda^5+a_5\lambda^4+a_6\lambda^3+a_7\lambda^2+a_8\lambda+a_9 \quad (30)$$

The ninth-degree characteristic polynomial is reduced using the approach of Podder and Gumel (2009), yielding the block representation in (31).

$$J(P^*)=\begin{bmatrix} J_{9\times4} & J^*_{9\times5} \end{bmatrix} \quad (31)$$

where:

$$\left(J_{9\times4}\right)=\begin{pmatrix} m_1 & -m_7 & 0 & \theta_1 \\ -m_8 & m_2 & -m_9 & 0 \\ 0 & m_{10} & m_3 & 0 \\ 0 & 0 & 0 & m_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mu_1 p_1 q_1 \xi_7 & 0 & 0 & 0 \\ 0 & \mu_2 p_2 q_2 \xi_8 & 0 & 0 \\ 0 & 0 & \mu_3 p_3 q_3 \xi_9 & 0 \end{pmatrix} \tag{32}$$

$$\left(J^{*}_{9\times5}\right)=\begin{pmatrix} 0 & 0 & -q_1 x & 0 & 0 \\ \theta_2 & 0 & 0 & -q_2 y & 0 \\ 0 & \theta_3 & 0 & 0 & -q_2 z \\ 0 & 0 & 0 & 0 & 0 \\ m_5 & 0 & 0 & 0 & 0 \\ 0 & m_6 & 0 & 0 & 0 \\ 0 & 0 & \mu_1\left(p_1 q_1 \xi_1 - c_1\right) & 0 & 0 \\ 0 & 0 & 0 & \mu_2\left(p_2 q_2 \xi_2 - c_2\right) & 0 \\ 0 & 0 & 0 & 0 & \mu_3\left(p_3 q_3 \xi_3 - c_3\right) \end{pmatrix} \tag{33}$$

Solving the two blocks in (32) and (33) separately gives the characteristic polynomial in (34).

$$\left(\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_3\right)\left(\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5\right) = 0 \tag{34}$$

The first factor of equation (34) is:

$$\left(\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_3\right) = 0 \tag{35}$$

where:

$$a_1 = \left(-m_4 - m_3 - m_2 - m_1\right)$$
$$a_2 = \left(m_2 m_1 + m_3 m_1 + m_4 m_1 + m_3 m_2 + m_4 m_2 + m_4 m_3 - m_8 m_7 + m_{10} m_9\right)$$
$$a_3 = \left(-m_1 m_2 m_3 - m_1 m_2 m_4 - m_1 m_3 m_4 - m_{10} m_9 m_1 - m_2 m_3 m_4 + m_3 m_7 m_8 + m_4 m_7 m_8 - m_4 m_9 m_{10}\right)$$
$$a_4 = m_4\left(m_1 m_2 m_3 + m_{10} m_9 m_1 - m_3 m_7 m_8\right)$$

The second factor of equation (34) is:

$$\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0 \tag{36}$$

$$\lambda^5 - \left(m_5 q_1 x q_3 z - \theta_3\theta_2 q_1 x\right)\lambda^2$$

$$a_1 = -\left(m_5 q_1 x q_3 z - \theta_3\theta_2 q_1 x\right) \text{ and } a_2 = a_3 = a_4 = a_5 = 0$$

Applying the Routh-Hurwitz criterion to the fifth-degree polynomial gives:

$$H_5 = \begin{pmatrix} a_1 & 1 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & 1 & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 \\ 0 & 0 & a_5 & a_4 & a_3 \\ 0 & 0 & 0 & 0 & a_5 \end{pmatrix} \tag{37}$$

$$a_i > 0, i = 1,2,3,4 \tag{38}$$

$$a_1 a_2 > a_2 a_3^2 \tag{39}$$

$$\left(a_1 a_2 a_3^{\,2} + a_1 a_3 a_5 + a_1 a_4 a_5 + a_2 a_3 a_5\right) > \left(a_1^{\,2} a_3 a_4 + a_1 a_2^{\,2} a_5 + a_3^{\,3} + a_5^{\,2}\right) \tag{40}$$

Therefore, the coexistence equilibrium point of the modified model is locally asymptotically stable whenever the conditions in inequalities (38), (39), and (40) are satisfied.

Numerical Simulation of the Model

The parameter values used in the numerical experiments are presented in Table 2.

Table 2. Baseline Parameter Values

Parameters	Baseline Values	Sources
γ_1	0.8	Mayengo et al. (2014)
γ_2	0.65	Mayengo et al. (2014)
γ_3	0.90	Umar and Musa (2017)
γ_4	0.90	Umar and Musa (2017)
γ_5	0.80	Assumed
γ_6	0.80	Assumed

k_1	600000	Mayengo et al. (2014)
k_2	500000	Mayengo et al. (2014)
k_3	600000	Mayengo et al. (2014)
k_4	500000	Mayengo et al. (2014)
k_5	600000	Assumed
k_6	500000	Assumed
β_1	0.000005	Mayengo et al. (2014)
β_2	0.000003	Mayengo et al. (2014)
β_3	0.0033-0.4	Assumed
β_4	0.0033-0.4	Assumed
d_1	0.2	Mayengo et al. (2014)
d_2	0.2	Mayengo et al. (2014)
d_3	0.2	Assumed
A	60000	Mayengo et al. (2014)
q_1	0.000005	Mayengo et al. (2014)
q_2	0.000012	Mayengo et al. (2014)
q_3	0.000005	Assumed
μ_1	0.1	Mayengo et al. (2014)
μ_2	0.12	Mayengo et al. (2014)
μ_3	0.12	Assumed
p_1	750	Mayengo et al. (2014)
p_2	700	Mayengo et al. (2014)
p_3	700	Assumed
c_1	500	Mayengo et al. (2014)
c_2	500	Mayengo et al. (2014)
c_3	500	Assumed
δ	0-0.9	Assumed
θ_1	0.5	Umar and Musa (2017)
θ_2	0.5	Umar and Musa (2017)
θ_3	0.5	Umar and Musa (2017)
σ_1	0.2	Umar and Musa (2017)
σ_2	0.2	Umar and Musa (2017)
σ_3	0.1	Umar and Musa (2017)
σ_4	0.1	Umar and Musa (2017)
σ_5	0.1	Assumed

σ_6	0.1	Assumed
E_1	150	Mayengo et al. (2014)
E_2	1.20	Mayengo et al. (2014)
E_3	1.20	Assumed

Experiment 1: Effect of Water-Pollution Control on the Unreserved-Zone Population

Using the baseline values in Table 2 and the initial condition $x(0) = 1000$, the water-pollution modification parameter δ is varied to assess the effect of pollution control.

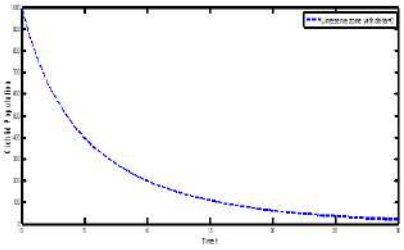


Figure 2. Effect of the water-pollution modification parameter at $\delta = 0$

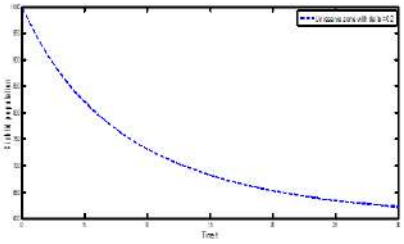


Figure 3. Effect of the water-pollution modification parameter at $\delta = 0.2$

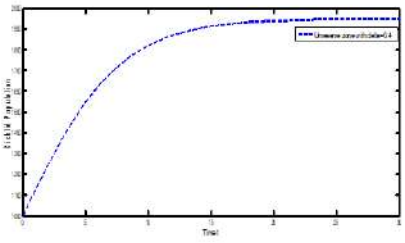


Figure 4. Effect of the water-pollution modification parameter at $\delta = 0.4$

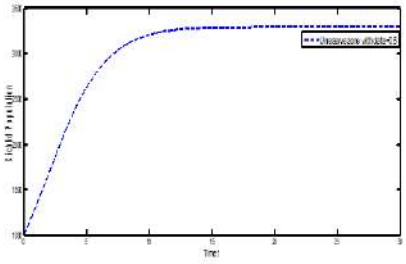


Figure 5. Effect of the water-pollution modification parameter at $\delta = 0.6$

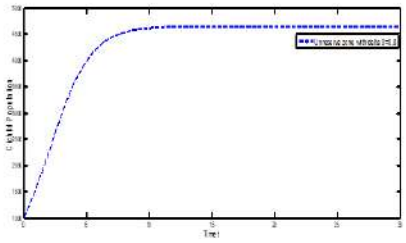


Figure 6. Effect of the water-pollution modification parameter at $\delta = 0.8$

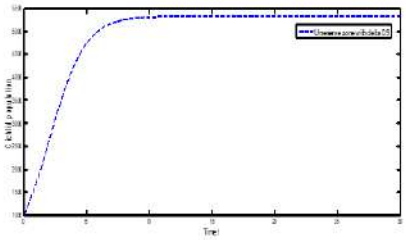


Figure 7. Effect of the water-pollution modification parameter at $\delta = 0.9$

Figures 2-7 illustrate the effect of the water-pollution modification parameter on the Cichlid population in the unreserved zone. When no pollution-control modification is applied (Figure 2), the population declines sharply. As the modification parameter increases from 0.2 to 0.9 (Figures 3-7), the decline is progressively reversed and the population approaches higher equilibrium levels. The simulation therefore indicates that improved water hygiene can substantially reduce pollution-induced mortality.

Experiment 2: Predation of Cichlid by Tilapia in the Unreserved Zone

Using the baseline values in Table 2 and the initial conditions $x(0)=1000,\ y(0)=700,$ the conversion coefficient β_2 for Tilapia predation on Cichlid is varied.

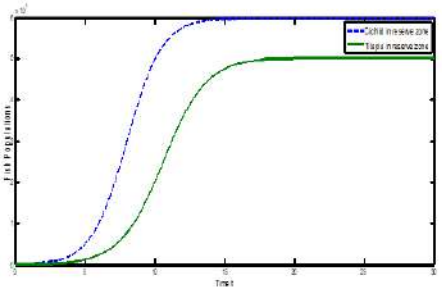


Figure 8. Predation of Cichlid by Tilapia at $\beta_2 = 0.003$

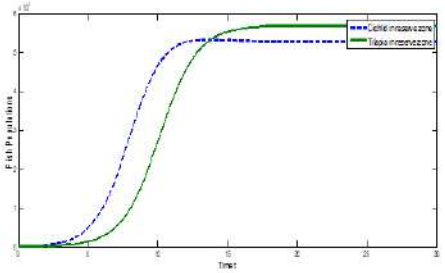


Figure 9. Predation of Cichlid by Tilapia at $\beta_2 = 0.1$

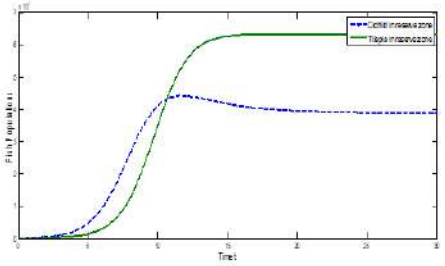


Figure 10. Predation of Cichlid by Tilapia at $\beta_2 = 0.2$

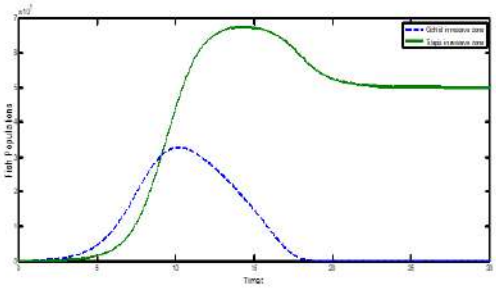


Figure 11. Predation of Cichlid by Tilapia at $\beta_2 = 0.3$

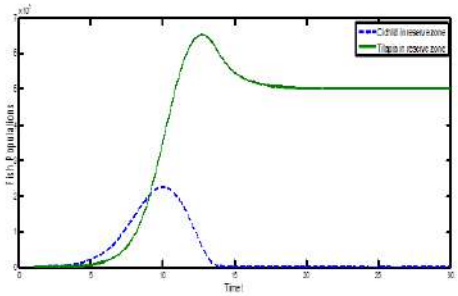


Figure 12. Predation of Cichlid by Tilapia at $\beta_2 = 0.4$

Figures 8-12 show how the conversion coefficient governing Tilapia predation affects the Cichlid population. As the coefficient increases, Tilapia gains more strongly from predation while the Cichlid population declines. In the highest-predation scenario, the Tilapia population peaks near t = 9 and the Cichlid population approaches extinction by approximately t = 13. The results demonstrate the potential for intense predatory pressure to eliminate the prey population.

Experiment 3: Combined Effects of Migration and Water-Pollution Modification

The simulation uses the baseline values in Table 2 and the initial conditions $x(0)=1000,$ $y(0)=700,$ and $z(0)=400;$. The migration parameter $\theta_i,\ i=1,2,3$ and the water-pollution modification parameter δ are varied jointly.

Experiment 3.1: Interaction Between Cichlid and Tilapia

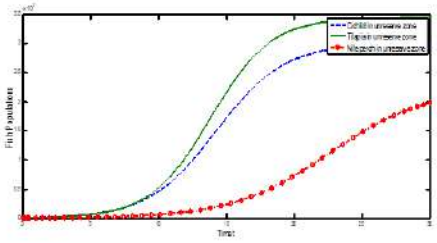


Figure 13. Combined intervention at $\theta_1=\theta_2=0,$ and $\delta=0$

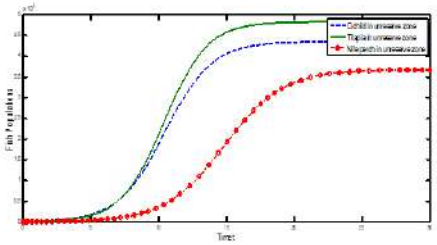


Figure 14. Combined intervention at $\theta_1=\theta_2=0,$ and $\delta=0.9$

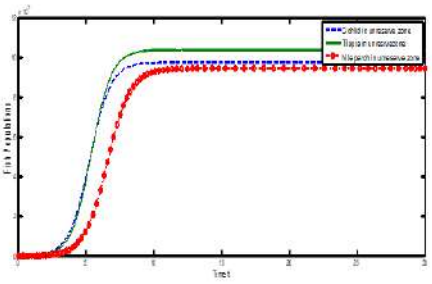


Figure 15. Combined intervention at $\theta_1=\theta_2=0.9,$ and $\delta=0$

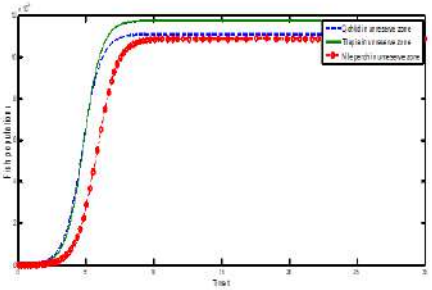


Figure 16. Combined intervention at $\theta_1=\theta_2=0.9,$ and $\delta=0.9$

Figures 13-16 illustrate the joint effects of migration and water-pollution modification on Cichlid and Tilapia in the unreserved zone under harvesting and pollution pressure. In Figure 13, both intervention parameters are zero, and Tilapia predation gradually reduces the relative abundance of Cichlid. Varying only one parameter produces a comparatively modest improvement (Figures 14 and 15). When both parameters are set to 0.9 (Figure 16), the populations increase markedly, particularly Tilapia. This outcome reflects the combined contribution of reserve-to-unreserved migration and reduced pollution-induced mortality.

Experiment 3.2: Interaction Between Tilapia and Nile Perch

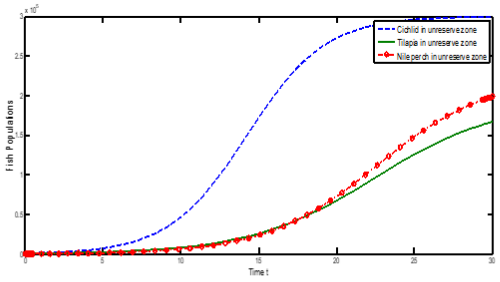


Figure 17. Combined intervention at $\theta_1 = \theta_2 = 0$, and $\delta = 0$

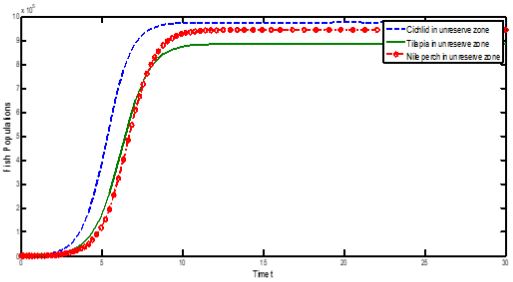


Figure 19. Combined intervention at $\theta_1 = \theta_2 = 0.9$, and $\delta = 0$

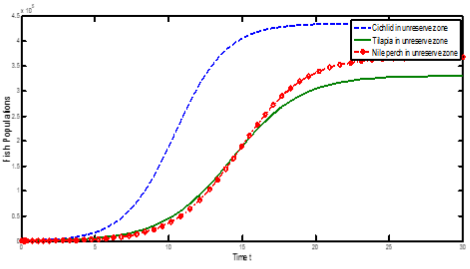


Figure 18. Combined intervention at $\theta_1 = \theta_2 = 0$, and $\delta = 0.9$

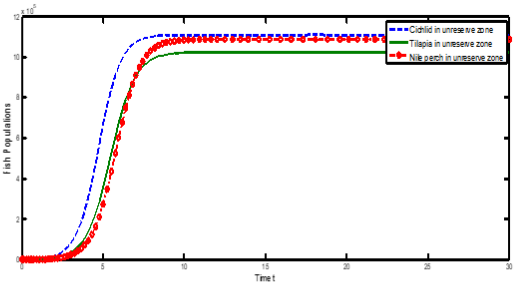


Figure 20. Combined intervention at $\theta_1 = \theta_2 = 0.9$, and $\delta = 0.9$

Figures 17-20 show the effects of migration and water-pollution modification on Tilapia and Nile perch in the unreserved zone. When both parameters are zero (Figure 17), strong Nile perch predation suppresses Tilapia and indirectly reduces Tilapia predation on Cichlid. Changing either migration or the pollution-modification parameter alone produces only a limited response (Figures 18 and 19). Applying both interventions at high levels (Figure 20) produces the greatest increase in population abundance and substantially offsets harvesting and pollution pressures. Predation between Nile perch and Tilapia nevertheless persists because it is an intrinsic ecological interaction.

DISCUSSION

The analysis considers several equilibria and establishes local stability conditions for two representative cases. The equilibrium point P_1 corresponds to persistence of the Cichlid population alone. Its local stability is established by transforming the Jacobian at P_1 into upper-triangular form and requiring all diagonal eigenvalues to have negative real parts. The

coexistence equilibrium point P^* is locally asymptotically stable when the Routh-Hurwitz conditions in inequalities (38), (39), and (40) are satisfied.

The numerical experiments examined the water-pollution modification parameter, Tilapia predation on Cichlid, Nile perch predation on Tilapia, and the combined use of reserve-to-unreserved migration and water-pollution control. These scenarios were designed to assess how the interventions reduce harvesting and pollution pressures while retaining biologically plausible prey-predator interactions.

Experiment 1 demonstrates that stronger water hygiene improves population persistence in the unreserved zone, as shown in Figures 2-7. Although Cichlid was used for illustration, the same mechanism applies to the other populations exposed to pollution-induced mortality. Experiment 2 confirms that increasing the conversion benefit associated with Tilapia predation can drive Cichlid towards extinction (Figures 8-12); an analogous mechanism applies to Nile perch predation on Tilapia. Experiment 3 shows that combining migration with water-pollution modification yields a stronger response than either strategy alone. The combined intervention reduces the adverse effects of harvesting and pollution and improves population abundance in the unreserved zone (Figures 13-20). Predation is not eliminated, however, because it remains a natural component of the ecosystem. Overall, the simulations support a combined management strategy rather than reliance on a single intervention.

CONCLUSION

This study developed and analysed a bioeconomic prey-predator model for Cichlid, Tilapia, and Nile perch in reserve and unreserved zones. A water-pollution modification parameter was introduced to represent water-hygiene measures in the unreserved zone. The analytical results established the existence of multiple equilibrium points and provided conditions for the local asymptotic stability of selected equilibria. Numerical simulations showed that pollution control improves population persistence, while the combined use of water hygiene and migration produces a stronger effect than either strategy alone. Predation continues under all scenarios because it is an inherent feature of prey-predator dynamics.

Future research may extend the model by incorporating stochastic dynamics, discrete-time formulations, agent-based methods, empirically estimated economic-rent functions, and calibration with observed fisheries data.

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