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Two Strains of Covid-19 Model with Vaccination and the Effect of Awareness Program on Its Control

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Abstract

The Covid-19 outbreak and the subsequent emergence of different viral strains posed a serious global public health challenge. This study proposes a mathematical model to analyze the transmission dynamics of two different Covid-19 strains and examine the effect of awareness on disease control. The study established the basic mathematical properties of the model, analyzed the disease-free and disease-endemic equilibria for both strains, conducted stability analysis, and computed the basic reproduction number, defined as $R_0 = \max(R_1, R_2)$. Stability conditions were examined for the strain-specific reproduction numbers, including cases in which $R_1 < 1$ while $R_2 > 1$ and $R_2 < 1$ while $R_1 > 1$. Numerical simulations were also conducted to support the analytical results and further illustrate the model dynamics. The findings show that increased awareness enhances vaccination uptake and reduces the basic reproduction number, thereby contributing to the control of disease transmission. The study concludes that awareness-based interventions play an important role in controlling the spread of Covid-19 strains through improved vaccination behavior and reduced transmission potential. These findings contribute to mathematical epidemiology by demonstrating the relevance of awareness-driven

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vaccination strategies in multi-strain infectious disease models and provide practical implications for public health authorities in strengthening awareness campaigns through media and social gatherings.

Keywords: Awareness; Basic Reproduction Number; Covid-19; Multi-Strain Model; Vaccination

Introduction

Covid-19 is the latest epidemic that ravaged the world. It is a viral disease caused by the "Severe Acute Respiratory Syndrome Corona virus 2" (SARS-COV-2 virus). The first detected case was in 'Wuhan', China in December 2019 [1]. The disease started as an outbreak of pneumonia of unknown cause and quickly became a disturbing epidemic which spread to over 210 countries across the globe causing serious burden to public health and socio-economic activities burden globally [2,3].

The disease became a source of serious concern not only because of the high mortality and transmission rate but also the rate at which it mutates. The new strains sometimes have characteristics different from existing ones in terms of the rates of transmission and death [4]. This has been a cause of serious worry as the development of drugs and vaccines have not proved to be capable of eliminating the disease. Covid-19 virus has various strains, in fact thousands of them after the first strain was discovered in the year 2019 [5].

Daniel (2020) model his work for the transmission of Covid-19 with non-linear forces of infection and the need for prevention measures in Nigeria [6]. Owhanda et. al. (2021) work was on the role of awareness in the control of Covid-19 in South-South Nigeria as presented in their paper titled "Awareness, Perception and Practices of Covid-19 prevention among residents of a state in South-South region of Nigeria [7]. Egunjobi and Makinde analysed two strains of Covid-19 disease without awareness. Musa et al carried out their work on the role of awareness program on the control of the pandemic. Their result shows that proper information can lead to the control of the disease [8].

This work is designed to study the dynamics of the spread two strains and study the role of awareness program in its control. This will be done by modifying the work of Musa et al, who worked on a single model with awareness.

The rest of this paper is structured as follows: The model is formulated in section 2, section 3 considers the model's well-possedness and boundedness. The analysis of the equilibrium points are presented in section 4 while the fifth section present the basic reproduction number and stability analysis. The model simulation and interpretation are presented in the sixth section, and the conclusion and recommendation are in section 7.

Materials and Methods

Our proposed model consist of seven mutually exclusive compartments. Table 1 gives the model State variables while table 2 gives the model parameters and their descriptions as used in this work.

Model Equations:

$$\frac{dS_u}{dt} = \Lambda - (\alpha_1 I_1 + \alpha_2 I_2 + \theta + \mu)S_u, \quad (1)$$

$$\frac{dS_a}{dt} = \theta S_u + \gamma R - \left(\frac{\alpha_1 I_1}{1+\theta I_1} + \frac{\alpha_2 I_2}{1+\theta I_2} + \omega + \mu\right) S_a, \quad (2)$$

$$\frac{dV}{dt} = \omega S_a - (\tau + \mu)V, \quad (3)$$

$$\frac{dI_1}{dt} = \frac{\alpha_1 I_1}{1+\theta I_1} S_a + \alpha_1 I_1 S_u - (\delta_1 + \beta_1 + d_1 + \mu)I_1, \quad (4)$$

$$\frac{dI_2}{dt} = \tau V + \frac{\alpha_2 I_2}{1+\theta I_1} S_a + \alpha_2 I_2 S_u - (\delta_2 + \beta_2 + d_2 + \mu)I_2, \quad (5)$$

$$\frac{dH}{dt} = \delta_1 I_1 + \delta_2 I_2 - (\phi + d_3 + \mu)H, \quad (6)$$

$$\frac{dR}{dt} = \beta_1 I_1 + \beta_2 I_2 + \phi H - (\gamma + \mu)R \quad (7)$$

with the initial conditions, $S_u(0) = \Lambda + S_{u,0}, Sa(0) = S_{a(0)}, V(0) = V_0, I_1(0) = I_{1,0}, I_2(0) = I_{2,0}, H(0) = H_0, R(0) = R_0$

Our model also assumed the following; that no individual is infected with both strains of the virus, there is possibility of reinfection after recovery and waning of the vaccine is also a possibility.

Table 1: Model State variables and their descriptions

State Variables	Description
S_u	Susceptible unaware population
S_a	Susceptible aware population
V	Vaccinated population
I_1	Population infected by first strain

I_2	Population infected by second strain
H	Hospitalised Population
R	Recovered population

Table 2: Model parameters and their descriptions

Parameter	Description
Λ	Recruitment rate into S_u
Θ	Awareness parameter
α_1	Infection rate of strain 1
α_2	Infection rate of strain 2
Ω	Vaccination rate of S_a
β_1	Natural recovery rate against I_1
β_2	Natural recovery rate against I_2
τ	Loss of immunity rate of vaccinated
δ_1	Rate at which I_1 get hospitalised
δ_2	Rate at which I_2 get hospitalised
γ	Rate at which recovered population move to S_a
$\emptyset$	Recovery rate of hospitalised population
ω	Vaccination rate
μ	Natural death rate
d_1	Death due to first strain
d_2	Death due to second strain
d_3	Death due to first or second strain

Positivity and Boundedness of the Solution

For the positivity of the model, we shall use the following theorem

Theorem1: Let $S_u(0) > 0, S_a(0) \geq 0, V(0) \geq 0, I_1(0) \geq 0, I_2(0) \geq 0, H(0) \geq 0, R(0) \geq 0 \in \Gamma$,

then the solution $S_u(t), S_a(t), V(t), I_1(t), I_2(t), H(t), R(t) \not< 0$, all $t > 0$.

Proof: To prove that $S_u(t) > 0$, we note from equation (1) that since Λ is positive the

$$\frac{dS_u}{dt} \geq -(\alpha_1 I_1 + \alpha_2 I_2 + \theta + \mu)S_u$$

$$lnS_u \geq -[\alpha_1 I_1 + \alpha_2 I_2 + \theta + \mu]t + k$$

Implies $$S_u(t) \geq e^{[\alpha_1 I_1 + \alpha_2 I_2 + \theta + \mu]t + k}$$

Now at t = 0, we obtain $S_u(0) > e^k$

Thus our general solution becomes

$$S_u(t) \geq S_u(0)e^{-[\alpha_1 I_1 + \alpha_2 I_2 + \theta + \mu]t}$$

Notice that $S(0) = e^k > 0$ and $e^{-[\alpha_1 I_1 + \alpha_2 I_2 + \theta + \mu]t} > 0$

Implies $S_u(0)e^{-[\alpha_1 I_1+\alpha_2 I_2+\theta+\mu]t} > 0$

Thus we have $S_u(t) > 0$ as reqiured

Same was done for other state variables.

Boundedness: In order to show that the model behave predictably and the solution do not grow without bound as time progresses, we need to show that it is bounded. This will be done by proving the following theorem.

Theorem 1: The model system is bounded in

$\Omega = \left\{S_u, S_a, V, I_1, I_2 H, R \in R_8 : N \leq \frac{\Lambda}{\mu} + (N(o) - \frac{\Lambda}{\mu})e^{-\mu t}\right\}$

Proof: The total population $N(t) = S_u(t) + S_a(t) + V(t) + I_1(t) + I_2(t) + H(t) + R(t)$ … (8)

Differentiating (16) w.r.t t, we've

$$\frac{dN}{dt} = \frac{dS_u}{dt} + \frac{dS_a}{dt} + \frac{dV}{dt} + \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dH}{dt} + \frac{dR}{dt} \ \ldots \qquad (9)$$

Substitute (1) – (7) into (9) and simplifying we have

$$\frac{dN}{dt} = \Lambda - \mu N - d_1 I_1 - d_2 I_2 - d_3 H \leq \Lambda - \mu N$$

Implies that $\frac{dN}{dt} \leq \Lambda - \mu N$ (10)

In order to find the solution set of (10), we first solve the differential equation

$$\frac{dN}{dt} = \Lambda - \mu N$$

$$\frac{dN}{dt} + \mu N = \Lambda \qquad (11)$$

Multiply (11) I.F.=$e^{\int \mu dt} = e^{\mu t}$ and solve to obtain $N(t) = \frac{\Lambda}{\mu} + ce^{-\mu t}$

With the initial condition $t = 0$ we have from (10), we have $N(t) \leq \frac{\Lambda}{\mu} + \left(N(0) - \frac{\Lambda}{\mu}\right) e^{-\mu t}$

Taking the limit as $t \to \infty$, we have $\lim_{t\to\infty} N(t) \leq \frac{\Lambda}{\mu}$

Thus,

$$0 \leq \frac{dN}{dt} \leq \left(N(0) - \frac{\Lambda}{\mu}\right) e^{-\mu t}$$

Also, taking the $limsup$ we have

$$\limsup_{t\to\infty} (S_u(t) + S_a(t) + V(t) + I_1(t) + I_2(t) + H(t) + R(t))$$

Hence the feasible region for the system is

$$\Omega = \left\{S_u(t) > 0, S_a(t) \geq 0. V(t) \geq 0, I_1(t) \geq 0, I_2(t) \geq 0, H(t) \geq 0, R(t) \geq 0 \text{ and } N(t) \leq \frac{\Lambda}{\mu}\right\}$$

Hence we conclude that the system is bounded in Ω.

Epidemiologically, it means the peak infection level is attained at $\frac{\Lambda}{\mu}$

Equilibriums

In this section we analyse the disease free, Strain1 disease endemic and Strain2 disease endemic equilibriums.

At equilibrium point(s) we have, $\frac{dS_u}{dt} = \frac{dS_a}{dt} = \frac{dV}{dt} = \frac{dI_1}{dt} = \frac{dI_2}{dt} = \frac{dH}{dt} = \frac{dR}{dt} = 0$

We shall use this fact to find the equilibrium points for our model

Disease Free Equilibrium DFE

At DFE, $I_1(t) = I_2(t) = 0$ as well as $H(t) = R(t) = 0$, while $S_u(t) \neq 0, S_a(t) \neq 0, V(t) \neq 0$

Thus DFE, $E^0 = \{S_u{}^0, S_a{}^0, V^0, 0,0,0,0\}$.

Now we solve (1)-(3) for S_u^0, S_a^0 and V^0 to obtain

$$E^0 = \left(\frac{\Lambda}{\theta+\mu}, \frac{\Lambda\theta}{(\theta+\mu)(\omega+\mu)}, \frac{\omega\theta\mu}{(\tau+\mu)(\theta+\mu)(\omega+\mu)}, 0,0,0,0\right)$$

Strain 1 Disease Endemic Equilibrium (DEE 1)

For strain1 DEE, $I_1 \neq 0, I_2 = 0$

The strain 1 DEE is given as $E^* = \{S_u{}^*, S_a{}^*, V^*, I_1{}^*, 0, H^*, R^*\}$

Next we shall express the state variables in terms of $I_1{}^*$ and the parameters and find a general solution to $I_1{}^*$ in terms of the parameters.

Solving $S_u{}' = 0$ from (1), i.e.

$$\Lambda - (\alpha_1 I_1 + \alpha_2 I_2 + \theta + \mu) S_u{}^* = 0$$

$$S_u{}^* = \frac{\Lambda}{\alpha_1 I_1 + \theta + \mu} \quad \ldots \qquad (12)$$

Doing same for (2) to (7), we have a quadratic equation in $I_1{}^*$ which can be expressed as

$$P_1(I_1^*)^2 + P_2(I_1^*) + P_3 = 0 \ldots \qquad (13)$$

Where

$P_1 = A_1 \alpha_1{}^2 = \alpha_1{}^2(\delta_1 + \beta_1 + d_1 + \mu)$

$P_2 = a_1[(\delta_1 + \beta_1 + d_1 + \mu)[(\omega + \mu) - \gamma(\theta + \mu) - \Lambda(\omega + \theta + \mu)]]$,

$P_3 = \alpha_1(\delta_1 + \beta_1 + d_1 + \mu)[(\omega + \mu) - \gamma(\theta + \mu) - \Lambda(\omega + \theta + \mu))]$.

Thus the general solution to (13) is given by

$$I_1^* = \frac{-P_2 \pm \sqrt{(P_2^2 - 4P_1P_3)}}{2P_1}$$

This solutions are $I_{1,1}^* = \frac{-P_2 + \sqrt{(P_2^2 - 4P_1P_2)}}{2P_1}, I_{1,2}^* = \frac{-P_2 - \sqrt{(P_2^2 - 4P_1P_3)}}{2a} < 0$

Observe that $I_{1,2}{}^*$ is not biologically feasible as the value will always negative

For $I_{1,1}^*$ to be biologically feasible, $\sqrt{P_2^2 - 4P_1P_3} > -P_2$

$$4P_1P_3 < 0$$

Thus, strain 1 disease endemic equilibrium is attained if the terms in $P_1 < 0$ or $P_3 < 0$, where P_1, P_2 and P_3 are functions of the parameters as described above.

Strain 2 Disease Endemic Equilibrium (DEE 2)

Strain 2 DEE occurs when $I_2 \neq 0, I_1 = 0$. It is given by

$$E^{**} = (S_u^{**}, S_a^{**}, V^{**}, I_1^{**}, 0, H^{**}, Q^{**})$$

To determine $S_u{}^{**}$, we equate $S_u{}' = 0$

$$S_u^{**} = \frac{\Lambda}{\alpha_2 I_2^{**} + \theta + \mu}$$

We do same for (2) to (7) and solve to obtain the polynomial

$$B_{20}(I_2^{**})^3 + B_{21}(I_2^{**})^2 + B_{22}(I_2^{**}) + B_{23} = 0 \qquad (14)$$

Where

$B_{20} = \alpha_2^2\theta(\delta_2 + \beta_2 + d_2 + \mu)(\eta + \mu)(\omega + \mu)$
$B_{21} = \frac{\gamma\alpha_2[(\tau+\mu)(\beta_2+\phi+\delta_2]\tau\omega+\gamma(\tau+\mu)[\tau+\mu)(\beta_2+\phi\delta_2]}{(\gamma+\mu)(\tau+\mu)}$
$B_{22} = (\theta + \mu)[\frac{\tau\omega\gamma(\tau+\mu+\phi\delta_2)}{(\gamma+\mu)(\tau+\mu)} + \theta\Lambda(\tau\omega\theta + (\tau + \mu)(\delta_2 + \beta_2 + d_2 + \mu))] + (\tau + \mu)(\omega + \mu)[\delta_2 + \beta_2 + d_2 + \mu + \alpha_2\Lambda]$
$B_{23} = \theta\Lambda\tau\omega - (\tau + \mu)(\theta + \mu)(\delta_2 + \beta_2 + d_2 + \mu)(\alpha_2 + \theta_2(\omega + \mu))$

In order to obtain the general solution of (14), we apply the Cardan's formula [9] for cubic equation to obtain the solution by following the steps below;

Step 1: we reduce the coefficient of $(I_2^{**})^3$ to unity to obtain

$$(I_2^{**})^3 + C_1(I_2^{**})^2 + C_2(I_2^{**}) + C_3 = 0 \ldots\ldots\ldots \quad (15)$$

Where $C_1 = \frac{B_{21}}{B_{20}}, C_1 = \frac{B_{22}}{B_{20}}, C_1 = \frac{B_{23}}{B_{20}}$

Step 2: We express (15) in the standard Cardian equation;

$$(I_2^{**})^3 + mI^{**} = n \quad (16)$$

Where $m = C_2 - \frac{C_1^2}{3}, n = -(\frac{2C_1^2}{27} - \frac{C_1C_2}{3} + C_3)$

Step 3: The Cardan solution to (16) is given as

$$I_2^{**} = \sqrt[3]{\frac{n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}} - \sqrt[3]{\frac{-n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}}$$

For the equation to be biologically feasible, $\frac{n^2}{4} \geq \frac{m^3}{27}$

Thus, strain 2 endemic equilibrium is attained if $\frac{n^2}{4} \geq \frac{m^3}{27}$

Basic Reproduction Number R_0 and Stability analysis

R_0 is a measure of the transmissibility of a viral disease and defined as the "expected number of secondary cases produced, in a completely susceptible population by a typical infected individual". It is a very important parameter in epidemiology. We shall use the "Next Generation Matrix" *NGM* method for its computation [10].

The infected compartments are equations (4) and (5). From where we the obtain new Infection for I_1 denoted as F_1 is given as

$$F_1 = \frac{\alpha_1 I_1}{1 + \theta I_1} S_a + \alpha_1 I_1 S_u \quad (15)$$

While the new Infection for I_2 denoted as F_2 is

$$F_2 = \frac{\alpha_2 I_2}{1 + \theta I_2} S_a + \alpha_2 I_2 S_u \quad (16)$$

The transition rates for I_1 and I_2 are respectively $V_1 = (\delta_1 + \beta_1 + d_1 + \mu)I_1$ and

$$V_2 = (\delta_2 + \beta_2 + d_2 + \mu)I_2$$

The Jacobian matrix $J(F)$ of the new infection F and $J(V)$ transition rates V are derived using $J(F) = \begin{pmatrix} \frac{\partial F_1}{\partial I_1} & \frac{\partial F_1}{\partial I_2} \\ \frac{\partial F_2}{\partial I_1} & \frac{\partial F_2}{\partial I_2} \end{pmatrix}$, $J(V) = \begin{pmatrix} \frac{\partial V_1}{\partial I_1} & \frac{\partial V_1}{\partial I_2} \\ \frac{\partial V_2}{\partial I_1} & \frac{\partial V_2}{\partial I_2} \end{pmatrix}$

$$J(F) = \begin{pmatrix} \frac{(1+\theta I_1)\alpha_1 S_a - \alpha_1 \theta I_1 S_a}{(1+\theta I_1)^2} + \alpha_1 S_u & 0 \\ 0 & \frac{(1+\theta I_2)\alpha_2 S_a - \alpha_2 \theta I_2 S_a}{(1+\theta I_2)^2} + \alpha_2 S_u \end{pmatrix}$$

At disease free i.e. $I_1 = I_2 = 0$, we have $J(F^0) = F$ as

$$F = \begin{pmatrix} \alpha_1 S_a + \alpha_1 S_u & 0 \\ 0 & \alpha_2 S_a + \alpha_2 S_u \end{pmatrix}$$

$$V = \begin{pmatrix} \delta_1 + \beta_1 + d_1 + \mu & 0 \\ 0 & \delta_2 + \beta_2 + d_2 + \mu \end{pmatrix}$$

The $K = FV^{-1} = \begin{pmatrix} \frac{\alpha_1(S_a+S_u)}{\delta_1+\beta_1+d_1+\mu} & 0 \\ 0 & \frac{\alpha_2(S_a+S_u)}{\delta_2+\beta_2+d_2+\mu} \end{pmatrix}$

Solving $det(K - \lambda I) = 0$, for λ, we have $\lambda = \frac{\alpha_1 \bar{S}}{\delta_1+\beta_1+d_1+\mu}, \frac{\alpha_2 \bar{S}}{\delta_2+\beta_2+d_2+\mu}$

The basic reproduction number R_0 is defined as the dominant *eigenvalue* for two strain disease given by $R_0 = max(R_1, R_2)$

Where $R_1 = \frac{\alpha_1 \Lambda(\omega+\mu+\theta)}{(\delta_1+\beta_1+d_1+\mu)(\theta+\mu)(\omega+\mu)}$, and $R_2 = \frac{\alpha_2 \Lambda(\omega+\mu+\theta)}{(\delta_2+\beta_2+d_2+\mu)(\theta+\mu)(\omega+\mu)}$

Stability Analysis: In order to determine the behaviour of the system in future, we to need to find the stability points. Stability analysis primarily focuses on the equilibrium point i.e. state where a system does not change with time. We shall use the Gershgorin Circle theorem to establish the local stability conditions [11, 12].

Theorem 3.6.1 *The Disease Free Equilibrium is locally asymptotically stable if* $R_0 < 1$ *and unstable if* $R_0 > 1$

Proof: For the prove we shall

i. derive the Jacobian matrix $J_{n,n}$ for the system

ii. derive the Jacobian matrix for the disease free

iii. compute the Gershgorin disc D_i, with centre c_i and radius r_i for i^{th} row (where $c_i = J_{i,i}$ and $r_i = \sum_{i \neq j} |J_{i,i}|$

iv. apply (iii) to the Jacobian matrix

v. the necessary condition for stability using the Gershgorin Circle theorem is $c_i + r_i < 0$.

The Jacobian matrix is given as

$$J = \begin{pmatrix} \frac{\partial S_u'}{\partial S_u} & \frac{\partial S_u'}{\partial S_a} & \frac{\partial S_u'}{\partial V} & \frac{\partial S_u'}{\partial I_1} & \frac{\partial S_u'}{\partial I_2} & \frac{\partial S_u'}{\partial H} & \frac{\partial S_u'}{\partial R} \\ \frac{\partial S_a'}{\partial S_u} & \frac{\partial S_a'}{\partial S_a} & \frac{\partial S_a'}{\partial V} & \frac{\partial S_a'}{\partial I_1} & \frac{\partial S_a'}{\partial I_2} & \frac{\partial S_a'}{\partial H} & \frac{\partial S_a'}{\partial R} \\ \frac{\partial V'}{\partial S_u} & \frac{\partial V'}{\partial S_a} & \frac{\partial V'}{\partial V} & \frac{\partial V'}{\partial I_1} & \frac{\partial V'}{\partial I_2} & \frac{\partial V'}{\partial H} & \frac{\partial V'}{\partial R} \\ \frac{\partial I_1'}{\partial S_u} & \frac{\partial I_1'}{\partial S_a} & \frac{\partial I_1'}{\partial V} & \frac{\partial I_1'}{\partial I_1} & \frac{\partial I_1'}{\partial I_2} & \frac{\partial I_1'}{\partial H} & \frac{\partial I_1'}{\partial R} \\ \frac{\partial I_2'}{\partial S_u} & \frac{\partial I_2'}{\partial S_a} & \frac{\partial I_2'}{\partial V} & \frac{\partial I_2'}{\partial I_1} & \frac{\partial I_2'}{\partial I_2} & \frac{\partial I_2'}{\partial H} & \frac{\partial I_2'}{\partial R} \\ \frac{\partial H'}{\partial S_u} & \frac{\partial H'}{\partial S_a} & \frac{\partial H'}{\partial V} & \frac{\partial H'}{\partial I_1} & \frac{\partial H'}{\partial I_2} & \frac{\partial H'}{\partial H} & \frac{\partial H'}{\partial R} \\ \frac{\partial R'}{\partial S_u} & \frac{\partial R'}{\partial S_a} & \frac{\partial R'}{\partial V} & \frac{\partial R'}{\partial I_1} & \frac{\partial R'}{\partial I_2} & \frac{\partial R'}{\partial H} & \frac{\partial R'}{\partial R} \end{pmatrix} \quad (17)$$

Applying (17) in (1)-(7), we have

$$J(E) = \begin{pmatrix} M_1 & 0 & 0 & -\alpha_1 S_u & -\alpha_2 S_u & 0 & 0 \\ \theta & M_2 & 0 & M_3 & M_4 & 0 & \gamma \\ 0 & \omega & -(\tau+\mu) & 0 & 0 & 0 & 0 \\ \alpha_1 I_1 & M_5 & 0 & P-A & 0 & 0 & 0 \\ \alpha_2 2_1 & M_6 & \tau & 0 & Q-B & 0 & 0 \\ 0 & 0 & 0 & \delta_1 & \delta_2 & -(\phi+d_3+\mu) & 0 \\ 0 & 0 & 0 & \beta_1 & \beta_2 & \phi & -(\tau+\mu) \end{pmatrix}$$

(18)

Where $M_1 = -(\alpha_1 I_1 + \alpha_2 I_2 + \theta + \mu)$, $M_2 = -\left(\frac{\alpha_1 I_1}{1+\theta I_1} + \frac{\alpha_2 I_2}{1+\theta I_2} + \omega + \mu\right)$, $M_3 = \frac{\alpha_1 S_a}{(1+\theta I_1)^2}$, $M_4 = \frac{\alpha_2 S_a}{(1+\theta I_2)^2}$, $M_5 = \frac{\alpha_1 I_1}{1+\theta I_1}$, $M_6 = \frac{\alpha_2 I_2}{1+\theta I_2}$,

$A = (\delta_1 + \beta_1 + \mu + d_1), B = (\delta_2 + \beta_2 + \mu + d_2), P = \frac{\alpha_1 S_a}{(1+\theta I_1)^2} + \alpha_1 S_u, Q = \frac{\alpha_2 S_a}{(1+\theta I_2)^2} + \alpha_2 S_u$

The Jacobian matrix $J(E^0)$ of the model at disease free equilibrium is obtained by substituting $I_1 = I_2 = 0$ into (18) to give

$$J(E^0) = \begin{pmatrix} -(\theta+\mu) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\omega+\mu) & 0 & 0 & 0 & 0 & \gamma \\ 0 & \omega & -(\theta+\mu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -A & 0 & 0 & 0 \\ 0 & 0 & \tau & 0 & -B & 0 & 0 \\ 0 & 0 & 0 & \delta_1 & \delta_2 & -(\phi+d_3+\mu) & 0 \\ 0 & 0 & 0 & \beta_1 & \beta_2 & \phi & -(\tau+\mu) \end{pmatrix}$$

Next we compute the discs $D_i's$

D_1: Centre $c_1 = -(\theta+\mu)$, radius $r_1 = |0|$

For stability, $-(\theta+\mu) < 0$

We do same for other rows and found that all $D_i's$ lie to the left half plane. Thus we have local stability at disease free equilibrium.

Strain 1 Disease Endemic Equilibrium E^* will be shown using the following theorem and applying the Gershgorin Circle theorem.

Theorem 3.6.2: The strain 1 disease endemic equilibrium E^* is locally asymptotically stable if $R_2 < 1$, $R_1 > 1$

Proof: In order to prove this theorem, we substitute $I_2 = 0$, into (18), while $I_1 \neq 0$ to obtain the following matrix

$$J(E^*) = \begin{pmatrix} -(\alpha_1 I_1+\theta+\mu) & 0 & 0 & -\alpha_1 S_u & 0 & 0 & 0 \\ \theta & -(\frac{\alpha_1 I_1}{1+\theta I_1}+\omega+\mu) & 0 & \frac{\alpha_1 S_a}{(1+\theta I_1)^2} & 0 & 0 & \gamma \\ 0 & \omega & -(\tau+\mu) & 0 & 0 & 0 & 0 \\ \alpha_1 I_1 & \frac{\alpha_1 I_1}{1+\theta I_1} & 0 & P-A & 0 & 0 & 0 \\ 0 & 0 & \tau & 0 & -B & 0 & 0 \\ 0 & 0 & 0 & \delta_1 & 0 & -(\phi+\mu+d_3) & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & \phi & -(\gamma+\mu) \end{pmatrix}$$

However, we shall also investigate the stability using the Gershgorin Circle theorem as above, to investigate the stability.

$$D_1: c_1 = -(\alpha_1 I_1+\theta+\mu), r_1 = \alpha_1 S_u, D_2: c_2 = -\left(\frac{\alpha_1 I_1}{1+\theta I_1}+\omega+\mu\right), r_2 = \frac{\alpha_1 S_u}{(1+\theta I_1)^2}$$

$$D_3: c_3 = -(\tau+\mu), r_3 = \omega, D_4: c_4 = P-A, r_4 = \alpha_1 I_1 + \frac{\alpha_1 I_1}{1+\theta I_1}, D_5: c_5 = -B, r_3 = \tau$$

$$D_6: c_6 = -(\emptyset+\mu+d_3), r_6 = \delta_1, D_7: c_7 = -(\gamma+\mu), r_7 = \beta_1 + \emptyset$$

We observe that all the disks except D_4satisfies the gergoshin condition. For $D_4$to satisfy the condition $P - A < 0$.

However, if $P - A > 0$ i.e $\frac{\alpha_1 S_a}{(1+\theta I_1)^2} + \alpha_1 S_u - (\delta_1 + \beta_1 + d_1 + \mu) > 0$ and $\theta = 0$ (no awareness on I_1), we have

$$\alpha_1 S_a + \alpha_1 S_u - (\delta_1 + \beta_1 + d_1 + \mu) > 0$$

$$\alpha_1 (S_a + S_u) > (\delta_1 + \beta_1 + d_1 + \mu)$$

$$\frac{\alpha_1 (S_a + S_u)}{(\delta_1 + \beta_1 + d_1 + \mu)} > 1$$

$$R_1 > 1$$

Thus, we conclude that strain 1 E^* is locally asymptotically stable if $R_2 < 1$ and $R_1 > 1$.

Same proof for strain 2 disease endemic equilibrium

Simulation & Interpretation of the Dynamics

Here we present some graphical representation of our system and some relationship between the variables contained in the model

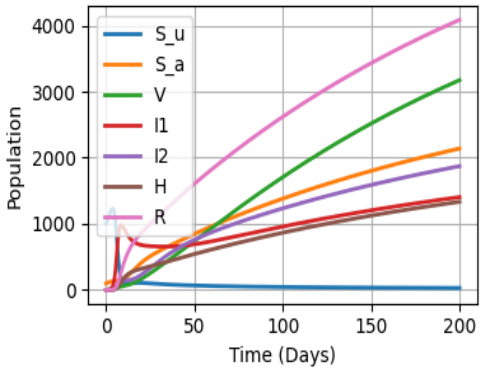


Figure 1: System dynamics

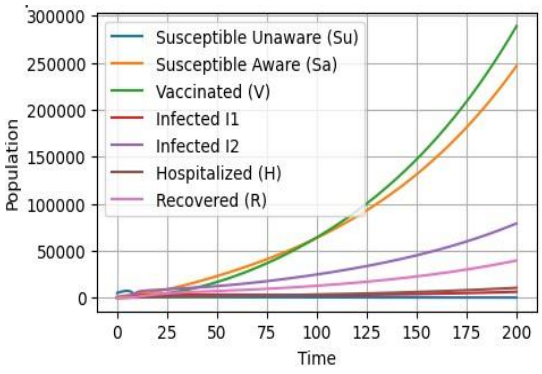


Figure 2: Influence of θ on the system

Figure 1 present the dynamics of the model which indicate a fall in susceptible unaware class as they move into the susceptible aware class and other classes with the highest being the recovery class. Figure 2 shows the influence of awareness on the system dynamics. We observe that awareness parameter θ only has significant effect on the susceptible aware, vaccinated and those who get effected with the second strain of the virus.

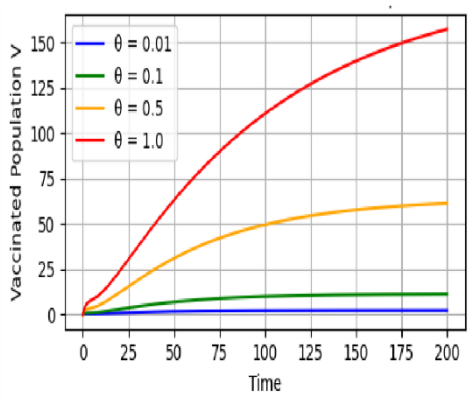


Figure 3: Effect of θ on Vaccination

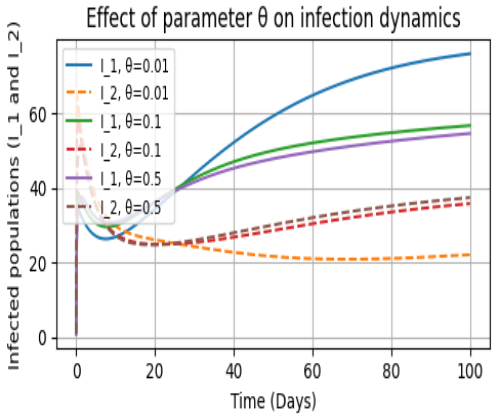


Figure 4: Effect of θ on I_1 and I_2

Figure 3 clearly indicates that as the awareness increases, the population of vaccinated individuals also increase at any particular time. Thus, indicating that awareness plays a positive role in vaccination. Figure 4 shows the influence of awareness on the infectious class. While awareness flattened the curve (reduces) infection of strain 2 with time, the population infected with strain 1 continue to grow with time.

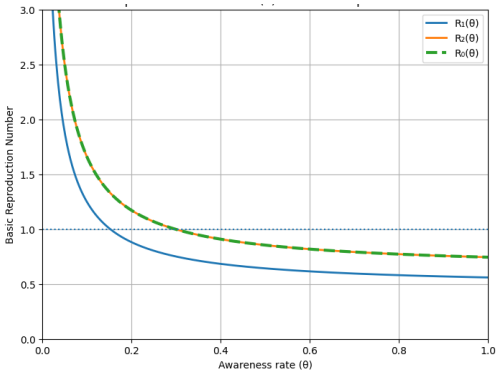


Figure 5: Effect of awareness on R_0

Figure 5 shows that as awareness has negative effect on the reproduction number. This indicates that increasing awareness will cause the disease to out. This is because more people will take precautionary measures that will reduce the disease transmission.

Conclusion

The study of two or more strains of Covid-19 is very relevant considering the fact that the disease mutate thus producing new strains. Awareness reduces infection rate and basic reproduction number. It also increases vaccination rate. Thus, we recommend that effective awareness campaign should be promoted in order to control the disease. This can

be done through the media, religious institutions and other social gatherings. Vaccines should also be made available and promoted.

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