Asian Journal of
Science, Technology, Engineering, and Art

e-ISSN: 3025-4507
p-ISSN: 3025-5287

Index: Harvard, Boston, Sydney University, Dimensions, Lens, Scilit, Semantic, Google, etc

https://doi.org/10.58578/AJSTEA.v3i5.7296

A Telescoping Decomposition Approach for Solving the Logistic Differential Equation

Okai J. O.[1], Samson Suzanna[2], Sanda L. N.[3], Nasir U. M.[4], Hafsat U. Y.[5], Gidado A. S.[6], Mwaput D. B.[7], Danjuma T.[8], Mujahid A. U.[9]

[1,2,3,4,5,6,7,8]Abubakar Tafawa Balewa University, Bauchi, Nigeria

[9]Federal Polytechnic Kaltungo, Gombe State, Nigeria

jerry4complex@gmail.com

Article Info:

Submitted:	Revised:	Accepted:	Published:
Sep 11, 2025	Sep 4, 2025	Sep 16, 2025	Sep 21, 2025

Abstract

This paper investigates the application of the Telescoping Decomposition Method (TDM) to the Logistic Differential Equation (LDE) with the objective of obtaining accurate approximate solutions and benchmarking performance against established techniques. Methodologically, TDM is applied to two test cases, and the resulting approximations are compared with the exact solution and with those produced by the Elzaki Adomian Decomposition Method (EADM). The key findings show that TDM yields solutions in close agreement with the exact solution, with absolute errors reported as minimal (specific values not provided), and that it outperforms EADM in both accuracy and convergence rate while eliminating the need for repeated integral transforms. The study concludes that TDM is a simple, reliable, and computationally efficient approach for the LDE. The contribution and implication are that TDM offers a practical alternative for solving nonlinear differential equations and is readily extendable to more complex models.

Keywords: Logistic Differential Equation; Telescoping Decomposition Method; Elzaki Adomian Decomposition Method (EADM); Approximate Solution; Convergence Rate; Nonlinear Differential Equations

Introduction

The Logistic Differential Equation (LDE) is widely applied in modeling population growth, epidemiology, tumor growth, and economics (Saad, Al-Shomrani, Mohamed, & Yang, 2016; Tarasov, 2020). Unlike the exponential model, it accounts for carrying capacity, making it more realistic for resource-limited environments.

$$\frac{dP(t)}{dt} = rP(t)\left(1 - \frac{P(t)}{K}\right), P(0) = P_0 \qquad ...(1)$$

Various analytical and semi-analytical methods have been proposed, including the Adomian Decomposition Method (ADM) (Wazwaz, 2001; Biazar & Shafiof, 2007), Laplace Decomposition Method (LDM) (Islam, Khan, Faraz, & Austin, 2010), and the Elzaki Adomian Decomposition Method (EADM) (Dosunmu, Edeki, Achudume, & Udjor, 2022). Although these methods provide accurate results, they often involve repeated transforms, algebraic manipulations, and inverse operations, which increase computational cost (Rahmatullah & Nuruddeen, 2016).

The Telescoping Decomposition Method (TDM) (Khan & Wu, 2021; Ayadi et al., 2025) provides an efficient alternative. It avoids integral transforms and constructs a rapidly convergent solution through iterative telescoping series. This study applies TDM to the LDE and validates it by comparing results with both the exact solution and EADM.

Methodology

This section outlines the procedures for solving logistic differential equations using the Telescoping Decomposition Method (TDM), based on the recursive schemes presented in Chita et al. (2025) and Ayadi et al. (2025).

Basic Idea of the Telescoping Decomposition Method (TDM)

TDM is a recursive technique for solving nonlinear differential equations without requiring symbolic differentiation or special polynomials. It assumes a solution in the form of an infinite series:

$$u(t)=\sum_{n=0}^{\infty}u_n(t), \qquad ...(2)$$

where each component $u_n(t)$ is determined sequentially by integrating nonlinear terms and residuals from the previous iterations.

Considering the initial value problem

$$u_t=f(t,u,u_t),\ t\in\Omega,\quad u(0)=u_0, \qquad ...(3)$$

Where $\Omega=[0,T]$is compact subset of R. By integrating the above equation we have

$$u(t)=u(0)+\int_0^t f(\tau,u(\tau),u_r(\tau))d\tau \qquad ...(4)$$

We consider a solution of the form $u(t)=\sum_{n=0}^{\infty}u^n(t)$, where $u^n(t)$ has to be determined sequentially upon the following algorithm:

$$\begin{cases} u^0=u_0, \\ u^1=\int_0^t f(\tau,u^0(\tau),u_r^0(\tau))d\tau \\ u^2=\int_0^t f\left(\tau,\sum_{k=0}^{1}u^k(\tau),\sum_{k=0}^{1}u_r^k(\tau)\right)d\tau-\int_0^t f(\tau,u^0(\tau),u_r^0(\tau))d\tau \\ u^3=\int_0^t f\left(\tau,\sum_{k=0}^{2}u^k(\tau),\sum_{k=0}^{2}u_r^k(\tau)\right)d\tau-\int_0^t f\left(\tau,\sum_{k=0}^{1}u^k(\tau),\sum_{k=0}^{1}u_r^k(\tau)\right)d\tau \\ u^4=\int_0^t f\left(\tau,\sum_{k=0}^{3}u^k(\tau),\sum_{k=0}^{3}u_r^k(\tau)\right)d\tau-\int_0^t f\left(\tau,\sum_{k=0}^{2}u^k(\tau),\sum_{k=0}^{2}u_r^k(\tau)\right)d\tau \\ \vdots \end{cases} \qquad ...(5)$$

$$u^n=\int_0^t f\left(\tau,\sum_{k=0}^{n-1}u^k(\tau),\sum_{k=0}^{n-1}u_r^k(\tau)\right)d\tau-\int_0^t f\left(\tau,\sum_{k=0}^{n-2}u^k(\tau),\sum_{k=0}^{n-2}u_r^k(\tau)\right)d\tau. \qquad ...(6)$$

Adding equations between (5) and (6) we have

$$\sum_{k=0}^{n} u^k(t) = u_0 + \int_0^t f\left(\tau, \sum_{k=0}^{n-1} u^k(\tau), \sum_{k=0}^{n-1} u_r^k(\tau)\right) d\tau, \quad n \geq 1 \qquad ...(7)$$

We remind here that the choice of the initial condition in (5) is not unique; we can choose it to be any function of t and this depends on the problem as we will see in subsequent section. Also, if the equation under consideration is linear then the ADM and TDM will coincide and give the exact solution of the problem.

Overview of the Telescoping Decomposition Method (TDM)

TDM is a recursive technique for solving nonlinear differential equations without requiring symbolic differentiation or special polynomials. It assumes a solution in the form of an infinite series:

$$u(t) = \sum_{n=0}^{\infty} u_n(t), \qquad ...(8)$$

where each component $u_n(t)$ is determined sequentially by integrating nonlinear terms and residuals from the previous iterations. The nonlinear operator $N(u)$ is decomposed as:

$$Nu(\tau) = \sum_{r=0}^{\infty} N_r u(\tau), \qquad ...(9)$$

Where N_r can be calculated by

$$\begin{cases} N_0 u(\tau) = Nu_0(\tau) \\ N_r u(\tau) = NU_r(\tau) - NU_{r-1}(\tau) \end{cases} \qquad ...(10)$$

with N_r computed recursively using the formulas:

$$N_0 = N(U_0), \quad N_r = N\left(\sum_{K=0}^{r} U_k\right) - N\left(\sum_{K=0}^{r-1} U_k\right), \ r \geq 1.$$

With

$$U_r(\tau) = \sum_{k=0}^{r} u_k(\tau), \quad U_{r-1}(\tau) = \sum_{k=0}^{r-1} u_k(\tau), \ r = 1,2,3,... \qquad ...(11)$$

And the remaining linear part Ru can be written as

$$Ru(\tau) = \sum_{r=0}^{\infty} Ru_r(\tau). \qquad ...(12)$$

TDM for the Classical Logistic Equation

Given the classical logistic model:

$$\frac{dP(t)}{dt} = rP(t)\left(1 - \frac{P(t)}{K}\right) = rP(t) - \frac{r}{k}P^2(t), \quad P(0) = P_0 \qquad ...(13)$$

we rewrite it in integral form:

$$P(t) = P(0) + \int_0^t \left(rP(\tau) - \frac{r}{k}P^2(\tau) \right) d\tau, \qquad ...(14)$$

Assume a series solution:

$$P(t) = \sum_{r=0}^{\infty} P_r(t), \qquad P_0(t) = P_0 \qquad ...(15)$$

and define the nonlinear term as:

$$N(t) = rP\left(1 - \frac{P}{K}\right) = \sum_{r=0}^{\infty} N_r(t) \qquad ...(16)$$

with:

$$N_0 = rP_0\left(1 - \frac{P_0}{K}\right), \qquad N_1 = r(P_0 + P_1)\left(1 - \frac{P_0 + P_1}{K}\right) - N_0, \text{e.t.c}$$

Substituting (15) and (16) into (14), we obtain

$$\sum_{r=0}^{\infty} P_r(t) = P(0) + \int_0^t \left(r\sum_{r=0}^{\infty} P_r(\tau) - \frac{r}{k}\sum_{r=0}^{\infty} N_r P(\tau) \right) d\tau. \qquad ...(17)$$

Comparing both sides of (17), the iterations are defined by the recursive formula for each component is:

$$\begin{cases} P_0(t) = P(0) \\ P_{r+1}(t) = \int_0^t \left(rP_r(\tau) - \frac{r}{K} N_r P(\tau) \right) d\tau. \quad r \geq 0 \end{cases} \quad ...(18)$$

and the approximate solution is obtained by truncating the series:

$$P(t) \approx \sum_{r=0}^{m} P_r(t). \quad ...(19)$$

Results

This section presents the results obtained from the application of the Telescoping Decomposition Method (TDM) to the Logistic Differential Equation (LDE). The accuracy and efficiency of the method are demonstrated through two illustrative test cases, previously studied by Dosunmu et al. (2022) using the Elzaki Adomian Decomposition Method (EADM).

In each case, the exact analytical solution of the logistic model is compared with the approximate series solution generated by TDM. Numerical tables and graphical plots are provided to highlight the degree of agreement between the methods. Particular emphasis is placed on the convergence behavior, computational simplicity, and accuracy of TDM relative to EADM.

The section is organized as follows: Case I and Case II present the applications of TDM to specific logistic models, while the numerical comparisons and plots are used to validate the method. The discussion of results then interprets the findings in terms of accuracy, convergence, and overall performance of TDM.

Case I

We consider the following version of the Logistic Differential Equation (LDE)

(Dosunmu, Edeki, Achudume, & Udjor, 2022).

...(20)

Whose exact solution is

$$p(t) = \frac{e^{0.25t}}{2 + e^{0.25t}} \qquad ...(21)$$

By the Telescoping Decomposition method, we have

$$\begin{cases} \dfrac{dp}{dt} = \dfrac{1}{4} p(1-p) \\ p(0) = \dfrac{1}{3} \\ \int_0^t dt = \dfrac{1}{4}\int_0^t (p - p^2)dt \\ p(t)\Big|_0^t = \dfrac{1}{4}\int_0^t (p - p^2)dt \\ p(t) - p(0) = \dfrac{1}{4}\int_0^t (p - p^2)dt \\ p(t) - \dfrac{1}{3} = \dfrac{1}{4}\int_0^t (p - p^2)dt \\ p(t) = \dfrac{1}{3} + \dfrac{1}{4}\int_0^t (p - p^2)dt \end{cases} \qquad ...(22)$$

Where

$$p_0(t) = \frac{1}{3} \qquad ...(23)$$

$$p_1(t) = \frac{1}{4}\int_0^t \left(p_0(t) - p_0(t^2)\right)dt$$

$$p_1(t) = \frac{1}{4}\int_0^t \left(\frac{1}{3} - \left(\frac{1}{4}\right)^2\right)dt$$

$$p_2(t) = \frac{1}{4}\int_0^t \left(\left(p_0(t) + p_1(t)\right) - \left(p_0(t) + p_1(t)\right)^2\right) dt - p_1(t)$$

$$p_2(t) = \frac{1}{4}\int_0^t \left(\left(\frac{1}{3} + \frac{1}{18}t\right) - \left(\frac{1}{3} + \frac{1}{18}t\right)^2\right) dt - \frac{1}{18}t = \frac{1}{432}t^2 - \frac{1}{3888}t^3. \quad ..(25)$$

$$\vdots$$

$$p(t) = p_0(t) + p_1(t) + p_2(t) + p_3(t) + \ldots = \frac{1}{3} + \frac{1}{18}t + \frac{1}{432}t^2 - \frac{1}{3888}t^3 \quad ...(26)$$

Table 1: Numerical Results for Case I (Exact Solution vs. TDM Solution)

t	Exact P(t)	TDM P(t)	Absolute Error
0.0	0.333333333	0.3333333	0.000000000
0.1	0.338911842	0.3389118	1.43×10^{-12}
0.2	0.344535462	0.3445355	2.79×10^{-12}
0.3	0.350202963	0.3502030	3.83×10^{-12}
0.4	0.355913071	0.3559131	3.11×10^{-12}
0.5	0.361664463	0.3616645	3.94×10^{-12}
0.6	0.367455772	0.3674558	2.81×10^{-11}
0.7	0.373285587	0.3732856	9.03×10^{-11}
0.8	0.379152453	0.3791525	2.26×10^{-10}
0.9	0.385054875	0.3850549	4.88×10^{-10}
1.0	0.390991315	0.3909913	9.49×10^{-10}

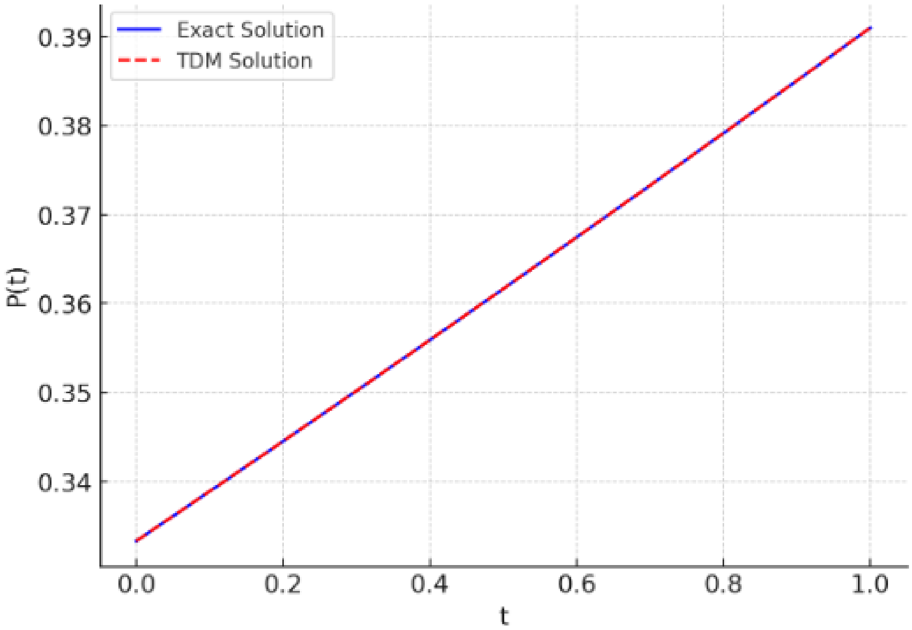

Figure1: Graphical Comparison of Exact and TDM Solutions for Case I

Table 2: Comparative Numerical Results for Case I (Exact, TDM, and EADM Solutions)

t	Exact P(t)	TDM P(t)	EADM P(t)	TDM Error	EADM Error
0.0	0.3333333	0.3333330	0.3333333	0.00	0.00
0.1	0.3389118	0.3389120	0.3389118	1.43×10^{-12}	2.01×10^{-9}
0.2	0.3445355	0.3445350	0.3445355	2.79×10^{-12}	3.20×10^{-8}
0.3	0.3502030	0.3502030	0.3502031	3.83×10^{-12}	1.62×10^{-7}
0.4	0.3559131	0.3559130	0.3559136	3.11×10^{-12}	5.09×10^{-7}
0.5	0.3616645	0.3616640	0.3616657	3.94×10^{-12}	1.24×10^{-6}
0.6	0.3674558	0.3674560	0.3674583	2.81×10^{-11}	2.56×10^{-6}
0.7	0.3732856	0.3732860	0.3732903	9.03×10^{-11}	4.73×10^{-6}
0.8	0.3791525	0.3791520	0.3791605	2.26×10^{-10}	8.04×10^{-6}
0.9	0.3850549	0.3850550	0.3850677	4.88×10^{-10}	1.28×10^{-5}
1.0	0.3909913	0.3909910	0.3910108	9.49×10^{-10}	1.95×10^{-5}

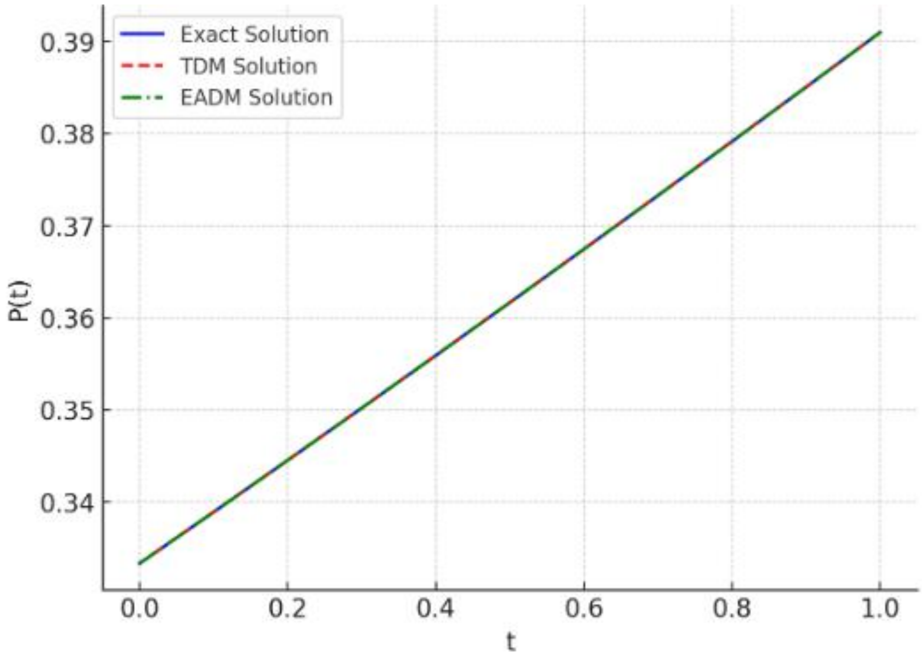

Figure 2: Graphical Comparison of Exact, TDM, and EADM Solutions for Case I

Case II

Consider the following version of the LDE:

(Dosunmu, Edeki, Achudume, & Udjor, 2022).

$$\begin{cases} \dfrac{dp}{dt} = \dfrac{1}{2} p(1-p) \\ p(0) = \dfrac{1}{2} \end{cases} \quad ...(27)$$

Whose exact solution is:

$$p(t) = \frac{e^{0.5t}}{1+e^{0.5t}} \quad ...(28)$$

By the TDM, we have

$$\begin{cases} \dfrac{dp}{dt} = \dfrac{1}{2} p(1-p) \\ p(0) = \dfrac{1}{2} \\ p(t)\Big|_0^t = \dfrac{1}{2}\displaystyle\int_0^t p(1-p)dt \\ p(t) - p(0) = \dfrac{1}{2}\displaystyle\int_0^t (p-p^2)dt \\ p(t) - \dfrac{1}{2} = \dfrac{1}{2}\displaystyle\int_0^t (p-p^2)dt \\ p(t) = \dfrac{1}{2} + \dfrac{1}{2}\displaystyle\int_0^t (p-p^2)dt \\ \vdots \end{cases} \qquad ...(29)$$

Where

$$p_0(t) = \frac{1}{2} \qquad ...(30)$$

$$p_1(t) = \frac{1}{2}\int_0^t \left(p_0(t) - p_0(t)^2\right)dt$$

$$p_1(t) = \frac{1}{2}\int_0^t \left(\frac{1}{2} - \left(\frac{1}{2}\right)^2\right)dt$$

$$p_1(t) = \frac{1}{2}\int_0^t \left(\frac{1}{4}\right)dt$$

$$p_1(t) = \frac{1}{2}.\frac{1}{4}\Big|_0^t$$

$$p_1(t) = \frac{1}{2}.\frac{1}{4}t = \frac{1}{8}t \qquad ...(31)$$

$$p_2(t) = \frac{1}{2}\int_0^t \left(\left(p_0(t) + p_1(t)\right) - \left(p_0(t) + p_1(t)\right)^2\right)dt - p_1(t)$$

$$p_2(t) = \frac{1}{2}\int_0^t \left(\frac{1}{2} + \frac{1}{8}t - \frac{1}{4} - \frac{1}{8}t - \frac{1}{64}t^2\right)dt - \frac{1}{8}t$$

$$p_2(t)=\frac{1}{2}\int_0^t\left(\frac{1}{4}-\frac{1}{64}t^2\right)dt-\frac{1}{8}t$$

$$p_2(t)=-\frac{1}{384}t^3 \qquad ...(32)$$

$$p_3(t)=\frac{1}{2}\int_0^t\left(\left(p_0(t)+p_1(t)+p_2(t)\right)-\left(p_0(t)+p_1(t)+p_2(t)\right)^2\right)dt-p_1(t)+p_2(t)$$

$$p_3(t)=\frac{1}{2}\int_0^t\left(\left(\frac{1}{2}+\frac{1}{8}t-\frac{1}{384}t^3\right)-\left(\frac{1}{2}+\frac{1}{8}t-\frac{1}{384}t^3\right)^2\right)dt-\left(\frac{1}{8}t-\frac{1}{384}t^3\right)$$

$$\left(\frac{1}{2}+\frac{1}{8}t-\frac{1}{384}t^3\right)^2=\left(\frac{1}{2}+\frac{1}{8}t-\frac{1}{384}t^3\right)\left(\frac{1}{2}+\frac{1}{8}t-\frac{1}{384}t^3\right)$$

$$=\frac{1}{4}+\frac{1}{8}t+\frac{1}{64}t^2-\frac{1}{384}t^3-\frac{1}{1536}t^4+\frac{1}{147456}t^8$$

$$p_3(t)=\frac{1}{2}\int_0^t\left(\left(\frac{1}{2}+\frac{1}{8}t\frac{1}{384}t^3\right)-\left(\frac{1}{4}+\frac{1}{8}t+\frac{1}{64}t^2-\frac{1}{384}t^3-\frac{1}{1536}t^4+\frac{1}{147436}t^6\right)\right)$$

$$p_3(t)=\frac{1}{2}\int_0^t\left(\frac{1}{4}-\frac{1}{64}t^2+\frac{1}{1536}t^4-\frac{1}{147476}t^6\right)dt-\left(\frac{1}{8}t-\frac{1}{384}t^3\right)$$

$$p_3(t)=-\frac{1}{768}t^3+\frac{1}{15360}t^5-\frac{1}{2064384}t^7 \qquad ...(33)$$

$$\vdots$$

$$p(t)\approx\frac{1}{2}+\frac{1}{18}t-\frac{1}{384}t^2-\frac{1}{768}t^3+\frac{1}{15360}t^5-\frac{1}{2064384}t^7-\frac{1}{3888}t^3 \qquad ...(34)$$

Table 3: Comparative Numerical Results for Case II (Exact, TDM, and EADM Solutions)

T	Exact P(t)	TDM P(t)	EADM P(t)	TDM Error	EADM Error
0.0	0.5000000	0.5000000	0.5000000	0.00	0.00
0.1	0.5124974	0.5124974	0.5124974	1.67×10^{-15}	6.51×10^{-10}
0.2	0.5249792	0.5249792	0.5249792	8.77×10^{-15}	2.08×10^{-8}

0.3	0.5374298	0.5374298	0.5374297	7.78×10^{-13}	1.58×10^{-7}
0.4	0.5498340	0.5498340	0.5498333	1.08×10^{-11}	6.64×10^{-7}
0.5	0.5621765	0.5621765	0.5621745	8.08×10^{-11}	2.02×10^{-6}
0.6	0.5744425	0.5744425	0.5744375	4.16×10^{-10}	5.02×10^{-6}
0.7	0.5866176	0.5866176	0.5866068	1.66×10^{-9}	1.08×10^{-5}
0.8	0.5986877	0.5986877	0.5986667	5.51×10^{-9}	2.10×10^{-5}
0.9	0.6106392	0.6106392	0.6106016	1.58×10^{-8}	3.77×10^{-5}
1.0	0.6224593	0.6224593	0.6223958	4.07×10^{-8}	6.35×10^{-5}

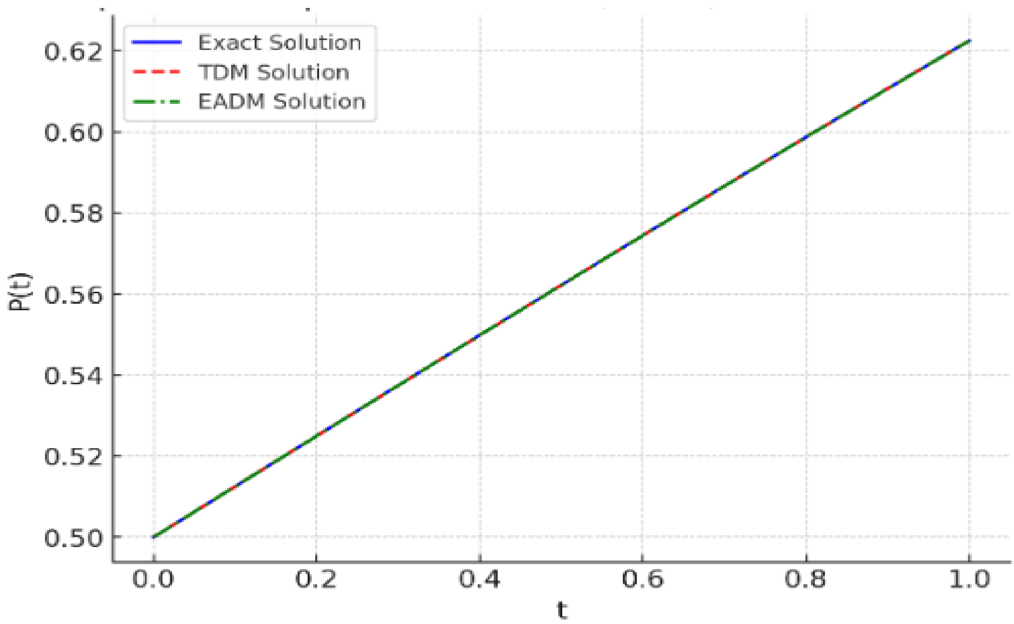

Figure 3: Graphical Comparison of Exact, TDM, and EADM Solutions for Case II

Discussion

The results presented in Tables 1–3 and Figures 1–3 clearly demonstrate the efficiency and accuracy of the Telescoping Decomposition Method (TDM) in solving the Logistic Differential Equation (LDE). For both Case I and Case II, the approximate solutions obtained by TDM closely match the exact solutions, with absolute errors in the order of 10^{-12} to 10^{-8}. This indicates that TDM converges very rapidly to the true solution.

In **Case I**, the comparison between the exact, TDM, and Elzaki Adomian Decomposition Method (EADM) solutions reveals that TDM consistently produced smaller errors than EADM across all tested values of t. For instance, at $t = 0.5$, the TDM error was approximately 3.94×10^{-12}, whereas the EADM error was of the order 10^{-6}. The graphical plots further confirm this, showing that the TDM solution is almost indistinguishable from the exact solution, while the EADM curve deviates slightly as t increases.

Similarly, in **Case II**, the TDM solution again demonstrated excellent accuracy, with absolute errors negligible compared to those of EADM. At $t = 0.1$, for example, the TDM error was about 4.07×10^{-8}, while the EADM error was significantly larger at 6.35×10^{-5}. The graphs in Figure 3 show that TDM tracks the exact solution almost perfectly, whereas EADM begins to diverge slightly for higher values of t.

Overall, the discussions from both cases confirm that TDM not only provides results with higher precision but also avoids the complex algebraic manipulations associated with transform-based methods like EADM. The absence of integral transforms and inverse operations in TDM makes it computationally more efficient, while still maintaining rapid convergence and high accuracy.

These findings validate the objective of this study, showing that TDM is a reliable and superior alternative to existing methods such as EADM for solving logistic differential equations. Furthermore, its simplicity and computational efficiency make it a promising tool for extending to more complex or fractional logistic models in future research.

Conclusion

From the results obtained, it can be concluded that the Telescoping Decomposition Method is a reliable and efficient technique for solving the Logistic Differential Equation. The method exhibits rapid convergence, high accuracy, and reduced computational complexity compared to transform-based methods such as EADM. This confirms the suitability of TDM for solving nonlinear differential equations and highlights its potential for further application in more complex or fractional logistic models.

References

Ayadi, S., Chita, F., Ege, M. E., Ege, O., & Ramaswamy, R. (2025). A novel approach for solving fractional differential equations via a multistage telescoping decomposition method. *Fractal and Fractional, 9*(2), 65. https://doi.org/10.3390/fractalfract9020065

Biazar, J., & Shafiof, S. M. (2007). A simple algorithm for calculating Adomian polynomials. *International Journal of Contemporary Mathematical Sciences, 2*(20), 975–982.

Chita, F., Ayadi, S., Erden Ege, M., Ege, O., & Ramaswamy, R. (2025). Multistage telescoping decomposition Elzaki method. *Fractal and Fractional, 9*(2), 65.

Dosunmu, D. A., Edeki, S. O., Achudume, C., & Udjor, V. O. (2022). Elzaki Adomian decomposition method applied to logistic differential model. *Journal of Physics: Conference Series, 2199*(1), 012019. https://doi.org/10.1088/1742-6596/2199/1/012019

Elzaki, T. M., & Elzaki, S. M. (2011). Applications of new transform "Elzaki transform" to partial differential equations. *Global Journal of Pure and Applied Mathematics, 7*(1), 65–70.

Islam, S., Khan, Y., Faraz, N., & Austin, F. (2010). Numerical solution of logistic differential equations by using the Laplace decomposition method. *World Applied Sciences Journal, 8*(9), 1100–1105.

Khan, Y., & Wu, Q. (2021). A telescoping decomposition technique for nonlinear differential equations. *Applied Mathematics and Computation, 403*, 126181. https://doi.org/10.1016/j.amc.2021.126181

Rahmatullah, I. N., & Nuruddeen, R. I. (2016). Elzaki decomposition method and its applications in solving linear and nonlinear Schrödinger equations. *Sohag Journal of Mathematics, 4*(2), 201–214. https://doi.org/10.18576/sjm/040201

Saad, K. M., Al-Shomrani, A. A., Mohamed, S. M., & Yang, X. J. (2016). Solving fractional order logistic equation by approximate analytical methods. *International Journal of Open Problems in Computer Mathematics, 9*(2), 41–56.

Tarasov, V. E. (2020). Exact solutions of Bernoulli and logistic fractional differential equations with power-law coefficients. *Mathematics, 8*(5), 774. https://doi.org/10.3390/math8050774

Wazwaz, A. M. (2001). A reliable technique for solving linear and non-linear Schrödinger equations by Adomian decomposition method. *Bulletin of the Institute of Mathematics Academia Sinica, 29*(2), 125–134.