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Use of Differential Equations in Population Growth Analysis

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Abstract

In this assignment we discussed differential equation applications in real-world scenarios. Differential equations are a significant area of mathematics with applications in many fields, including engineering, astronomy, biology, chemistry, physics, economics, and finance. Differential equations are used in economics to explain many concepts such as restricted and infinite growth, equilibrium and stability, and the dynamics of supply and demand in an economy. Differential equations will be used to examine several facets of Nepal in this research assignment.

Keywords: Differential equations, ordinary differential equation, partial differential equation

Introduction

The development of calculus by Gottfried Wilhelm Leibniz and Isaac Newton in the 17th century gave rise to differential equations. For characterizing the rates of change in physical systems, they become indispensable. In the eighteenth century, the study of ordinary differential equations was furthered by Leonhard Euler and Joseph-Louis Lagrange, while the significance of partial differential equations was brought to light by Joseph Fourier's research on heat conduction. Mathematicians such as Henri Poincaré and Carl Gustav Jacobi carried out additional progress in the 19^{th} century. More recent developments have concentrated on numerical solutions and applications in a variety of scientific domains.

An equation that Involves independent and dependent variables together with the derivatives of dependent variable with respect to independent variable is called a differential equation. If the function, whose derivatives are used in the equation, is of a single independent variable then it is called an ordinary differential equation (ODE).

Example: $\frac{dy}{dx} + y = 0$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

When an equation contains the derivative of a function with respect to two or more than two independent variables then it is called a partial differential equation (PDE).

The order of the highest derivative of the function present in the differential equation, is known as order of the differential equation.

The degree of the highest derivative of the function present in the differential equation after making free from function and radical is known as degree of the differential equation.

Example: $\left(\frac{dy}{dx}\right)^{15} + \left(\frac{d^2y}{dx^2}\right)^{12} + \frac{d^3y}{dx^3} = 0$

Therefore, order =3 and degree =1

A solution is a relation between the variables which is free from derivatives and satisfy the given differential equation.

If a solution consists of the number of arbitrary constants equal to the order of the differential equation, then the solution is known general solution.

The solution obtained from general solution by giving particular values to one or more of constants is known as particular Solution.

Example: y=cx is the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$.

If we set the condition y(1) = 1 then the solution becomes y = x which is the particular solution of $\frac{dy}{dx} = \frac{y}{x}$.

Definitions

Differential equations: An equation that involves independent and dependent variables together with the derivatives of dependent variable with respect to independent variable is called a differential equation.

Ordinary differential equation: If the function, whose derivatives are used in the equation, is of a single independent variable then it is called an ordinary differential equation (ODE).

Partial differential equation : When an equation contains the derivative of a function with respect to two or more than two independent variables then it is called a partial differential equation (PDE).

Problem 1:

According to the census of 2021A D, the annual average population growth rate of Nepal is 0.92% per annum. How long does it take for the population to be doubled?

Solution 1:

Given that,

Rate = 0.92% i.e. 0.0092

Let us suppose that the initial population of Nepal be P_0 $at\ t = 0.$

Using the formula for exponential growth,

$P(t) = P_0 \,.\, e^{rt}$

For t when the population will be doubled i.e. $P(t) = 2P_0$

Substitute this into formula,

$2P_0 = P_0 \,.\, e^{0.0092t}$

Or, $2 = e^{0.0092t}$

Or, $e^{0.0092t} = 2$

Or, $0.0092t = \log_e 2$

Or, $t = \frac{1}{0.0092} * 0.693147$

Or, t = 75.34

Therefore, it will take 75.34 years to get the population doubled.

Problem 2:

According to the census of 2021A D, the annual average inflation rate of Nepal is 6.32% per annum. How long does it take for the inflation to be doubled?

Solution 2:

Given that,

Rate = 6.32% i.e. 0.0632

Let us suppose that the initial inflation of Nepal be $I_0\ at\ t = 0.$

Using the formula for exponential growth,

$I(t) = I_0 . e^{rt}$

For t when the inflation will be doubled i.e. $I(t) = 2I_0$

Substitute this into formula,

$$2I_0 = I_0 . e^{0.0632t}$$

Or, $2 = e^{0.0632t}$

Or, $e^{0.0632} = 2$

Or, $0.0632t = \log_e 2$

Or, $t = \frac{1}{0.0632} * 0.693147$

Or, t = 10.96

Therefore, it will take 10.96 years to get the inflation doubled.

Problem 3 :

According to the census of 2011, the population density of Nepal was 180 person per square kilometre which is now 198 person per square kilometre according to the census of 2021. Find the population density growth rate of Nepal?

Solution 3:

Given that,

t= 10 years

P(t) = 198

$P_0 = 180.$

Using the formula for exponential growth,

$P(t) = P_0 \,.\, e^{rt}$

$or, 198 = 180 \,.\, e^{r10}$

$or, \frac{198}{180} or, 1.1 = e^{r10} = e^{r10}$

$or, e^{r10} = 1.1$

$or, r10 = \log_e 1.1$

$or, 10r = 0.09531$

$or, r = 0.009531$

To express the growth rate as a percentage,

r = 0.009531*100

I.e. r = 0.9631%

Therefore, the rate of growth of the population density is approximately 0.9531% per annum.

Problem 4:

The GDP of Nepal was Rs. 4851.62 billion in FY 2021/22 and the annual growth rate was 5.8% .

What will be the GDP of Nepal after 10 years?

Solution 4:

Given that,

Rate = 5.8 % i.e. 0.058

$P_0 = Rs\ 4851.62\ billion$

T= 10 years

Using the formula for exponential growth,

$P(t) = P_0 . e^{rt}$

$or, P(t) = 4851.62 * \mathrm{e}^{0.058*10}$

$or, P(t) = 4851.62 * \mathrm{e}^{0.58}$

$or, P(t) = 4851.62 * 1.7860$

$or, P(t) = 8664.99332$

$\approx P(t) = 8665$

Therefore, the GDP of Nepal will be approximately Rs. 8665 billon after 10 years.

Problem 5

The total number of tourist arrivals in Nepal increased significantly to 3,70,906 in 2021/22, which was

70,123 in the previous fiscal year. Find the rate of increment?

Solution 5

Given that,

T= 1 year

T(t) = 370906

$T_0 = 70123$

Using the formula for exponential growth,

$T(t) = \mathrm{T}_0 . \mathrm{e}^{\mathrm{rt}}$

$or, 370906 = 70123 * e^{r*1}$

$or, 370906 = 70123 * e^{r}$

$or, \frac{370906}{70123} = e^{r}$

$or, e^{r} = \frac{370906}{70123}$

$or, e^{r} = 5.2893$

$or, r = \log_e 5.2893$

$or, r = 1.665$

To express the growth rate as a percentage,

r = 1.665*100

I.e. r = 166.5%

Therefore, the rate of increment in number of tourists in Nepal is 166.5%.

Problem 6:

The population of Yadav was 1054458 in 2011 which has reached to 1228581 in the census of 2021. Find the rate of growth in Yadav population?

Solution 6:

Given that,

P(t) = 1228581

$P_0 = 1054458$

T= 10 years .

Using the formula for exponential growth,

$P(t) = P_0 . e^{rt}$

$or, 1228581 = 1054458 . e^{r*10}$

$$or, \frac{1228581}{1054458} = e^{r*10}$$

$$or, 1.165 = e^{r*10}$$

$$or, e^{r*10} = 1.165$$

$$or, 10r = \log_e 1.165$$

$$or, 10r = 0.15$$

$$or, r = 0.015.$$

To express the growth rate as a percentage,

r = 0.015*100

I.e. r = 1.5%

Therefore, the rate of growth in Yadav population is 1.5 %.

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