

## Modern Designs Using Parabolic Curves as a New Paradigm for Sophisticated Architecture

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### Abstract

This research investigates the spatial aspects of architecture through the practical use of parabolic curves, suggesting that these curves are an architectural advancement in the direction of the creation of complex, efficient and beautiful buildings. Focused on material properties and environmental concerns, as well as the strength of the structure, the study explores the use of parabolic geometry in modern architecture. The integration of case studies, computational modeling, and theoretical analysis is done in an interdisciplinary context. The study comprises: 1) Theoretical Framework: A survey of the geometrical and historical importance of parabolas in architectural design; 2) Computational Modeling: The processes of design tools for parabolic structures and evaluation of their load, material, and aesthetic efficiency; 3) Case Studies: Studies of modern buildings that include parabolic curves in their designs, including well-known buildings and new tendencies in Eco-Architecture.

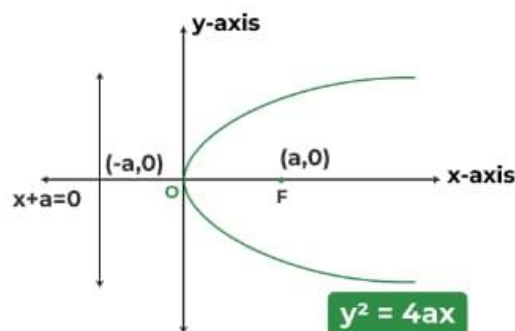
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## Introduction

Historical Background: Because of the beauty and structural positive attributes associated with parabolic shapes, they have been largely used in architecture, both in the past and at the present times. High and low parabolic arches were also utilized in the ancient Roman as well as the Islamic architecture for structural stability and beauty. While Baroque architects like Bernini used these curves for effects, Renaissance figures like Leonardo da Vinci studied these curves for their geometric properties. During the Industrial Revolution, large span constructions notably the Eiffel Tower as well as bridges used parabolic arches. The Chicago style of modern architecture which was one of the first design movements to use parabolas in modernism and the works of architects such as Frank Lloyd Wright including the Guggenheim Museum are cited where the parabolic forms were well adopted. With the advancement of today's design techniques and materials, parabolic curves are used in other complexities in structures such as the Sydney Opera house and Eden Project where the parabolic curves basic design features still inspire novel sustainable architectural designs.

## Definitions

- **Parabolic Curve:** ( $y = ax^2 + bx + c$ ) is a quadratic equation that defines a curve. It is a U-shaped curve in architecture that is used to distribute loads uniformly in structural elements like arches and roofs.



- **Structural Efficiency:** - Making the best use of resources and design to produce the intended results with the least amount of waste. In order to reduce material requirements and preserve stability while distributing loads evenly, parabolic curves improve structural efficiency.

- **Load Distribution:** - The process of allocating weights and forces throughout a structure. By guiding forces via the curve, parabolic curves efficiently control load distribution and contribute to the preservation of structural integrity.
- **Aesthetic Appeal:** - A design's visual appeal. Because of their graceful and harmonic patterns, parametric curves are prized for adding to the overall beauty of architectural forms.
- **Material Optimization:** - The art of utilizing materials to optimize efficiency and reduce waste. Less material can be used while still getting the required strength and usefulness thanks to parabolic curves.
- **Parametric Design:** A design approach that defines and manipulates complicated forms, such as parabolic curves, using computer tools and algorithms. Precise and creative architectural solutions are possible with this method.
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## Discussions

Through the use of parabolic curves which are known as shapes that are clean and smooth, the project aims at not only reinterpreting but redefining modern architecture. This provides a paradigm shift in architectural practice that emphasizes not only efficiency but also the beauty of the structures that is brought out by a fusion of form and function.

### What is the use of parabolic curve in architecture?

Parabolic curves due to their structural efficiency and aesthetic satisfaction are widely utilized in architecture. They are best designs for large spans with less material like arches, domes, and bridges since they transfer load evenly. These curves also provide exciting shapes adding values to the design as can be seen in historical structures like the Gateway Arch. Such geometric shapes in roofs and larger free spaces enable architects to create light and spacious buildings while require less number of supporting columns which stabilizes the building.

### Problems based on real life applications of parabolic curves:

#### Why are parabolic curve used to construct bridges?

That is because adopting parabolic curve on bridge making works to alleviate forces thus ensuring effective load distribution. Parabolic shape as we already know, works the way you expect in terms of compaction of forces, reduces the bending moments, and ensures stress management in a given cross sectional area, which brings us to maximum compression.

#### Problem 1:

**An arch bridge has a parabolic shape. The arch is 80 feet wide and reaches to a maximum height of 40 feet. What is the height of arch 15 feet away from center of the base?**

#### Solution:

The general equation of parabola is

$$y=a(x-h)^2+k$$

Where,

- (h, k) are the vertex of the parabola

Here, the vertex is at the top of the arch, which is the point (0, 40), since the centre of the arch is at  $x=0$  and the maximum height is at 80 feet.

So,  $h=0$  and  $k=40$ , making the equation:

$$y= ax^2+40$$

Now,

The total width of the arch is 80 feet, so the horizontal distance from the center of the arch to either side is 40 feet (i.e. half of the width). At this point,  $y=0$  into the equation to find a:

$$0= a (40)^2+40$$

$$0=1600a+40$$

$$1600a=-40$$

$$a=-\frac{1}{40}$$

So, the equation of parabola is:

$$y = -\frac{1}{40}x^2 + 40$$

Again,

to find height of the arch 15 feet away from the center, substituting  $x=15$  into the equation:

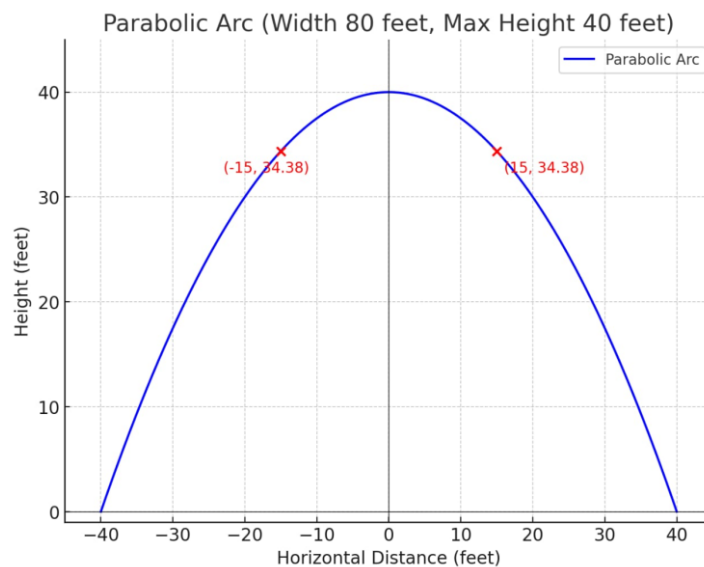
$$y = -\frac{1}{40}(15)^2 + 40$$

$$y = -\frac{1}{40}(225) + 40$$

$$y = -\frac{225}{40} + 40$$

$$y = -5.625 + 40$$

$$y = 34.375 \text{ feet}$$



**Problem 2:**

A parabolic arch bridge has a width of 100 feet and reaches a height of 45 feet at its midpoint. Find the height of the bridge 25 feet from the center.

**Solution:**

Here,

The arch is symmetric about the center. The total width of parabolic arch bridge is 100 feet.

Therefore, the arch extends from -50 feet to 50 feet from the center with the highest point i.e. vertex being 45 feet above the ground at the center.

The general equation of parabola is

$$y=a(x-h)^2+k$$

Where,

- $(h, k)$  are the vertex of the parabola

Here, the vertex is at the top of the arch, which is the point  $(0, 45)$ , since the centre of the arch is at  $x=0$ .

So,  $h=0$  and  $k=45$  making the equation

$$y=-ax^2+45$$

Now,

The parabola passes through one end of the arch at  $(50, 0)$ :

$$0=-a(50)^2+45$$

$$0=-2500a+45$$

$$2500a=45$$

$$a=\frac{45}{2500}$$

$$a=\frac{9}{500}$$

So, the equation of the parabola is:

$$y=-\frac{9}{500}x^2+45$$

Substituting  $x=25$  into the equation  $y=-\frac{9}{500}(25)^2+45$

$$y=-\frac{9}{500}\times 625+45$$

$$y=-\frac{5625}{500}+45$$

$$y=-11.25+45$$

$$y=33.75 \text{ feet}$$

So, the height of bridge 25 feet away from the center is 33.75 feet.

### Why is parabolic curve used in making tunnels and overpasses?

Parabolic curves are the design patterns for boring tunnels and constructing flyovers. This is because they assist in changing heights very smoothly leading to better safety, visibility and comfort for the driver. Such curves also allow for an even distribution of various vertical loads imposed on the structure, and better water runoff as well, making the whole system more effective and long-lasting.

#### Problem 1:

A tunnel has a parabolic arch described by the equation  $y = -ax^2 + c$ , where  $c$  is the maximum height at the center (vertex) of the arch. If the tunnel is 20 feet wide at the base and the maximum height is 15 feet, what is the value of  $a$ ?

#### Solution:

The parabolic equation for the tunnel:

$$y = -ax^2 + c$$

Given:

The tunnel is 20 feet wide at the base, meaning the total width from one side to the other is 20 feet. Therefore, the  $x$ -values range from -10 to 10 (centered at the origin).

The maximum height (vertex)  $c=15$  feet. The height of the tunnel at the edges (where  $x=-10$  and  $x=10$ ) will be 0 (the height of the ground).

Using the equation at  $x=10$ :

$$0 = -a(10^2) + 15$$

This simplifies to:

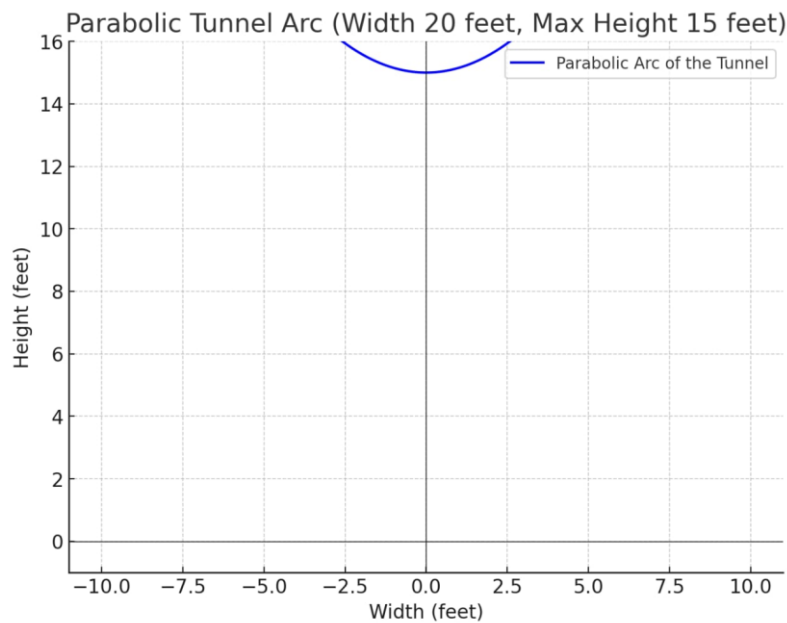
$$0 = -100a + 15$$

$$100a = 15$$

Thus,

$$a = \frac{15}{100} = 0.15$$

Therefore, the value of  $a$  is 0.15.



**Problem 2:**

A tunnel is shaped like a parabola given by  $y=0.01(x-30)^2$ . What is the horizontal distance between the points where the tunnel reaches a height of 3 feet?

**Solution:**

The parabolic equation given for the tunnel:

$$y = 0.01(x - 30)^2 + 2$$

To find the horizontal distance between the points where the tunnel reaches a height of 3 feet,

$$3 = 0.01(x - 30)^2 + 2$$

$$1 = 0.01(x - 30)^2$$

$$100 = (x - 30)^2$$

Taking the square root of both sides:

$$x - 30 = \pm 10$$

For the positive case:

$$x - 30 = 10$$

$$x=40$$

For the negative case:

$$x - 30 = -10$$

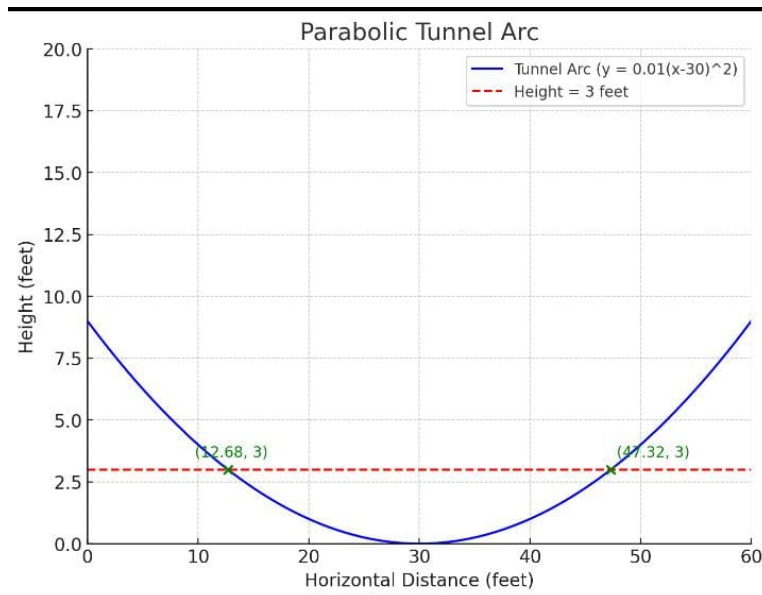
$$x = 20$$

6. The points where the height is 3 feet are  $x= 20$  and  $x=40$ .

7. The horizontal distance between these points is:

$$40 - 20 = 20 \text{ feet}$$

Thus, the horizontal distance between the points where the tunnel reaches a height of 3 feet is 20 feet.



### Parabolic curve in roof Structures:

- Stadiums and Arenas: Parabolic shapes are used in the roofs of stadiums and large halls to create expansive, column-free spaces. This allows for a larger spectator area without obstructive supports.
- Shell Structures: Architects often use thin-shell structures with parabolic curves, such as in airport terminals and auditoriums, to cover large areas with minimal material, as seen in structures like the Sydney Opera House.

### **Some modern examples of Parabolic curve architecture:**

Parabolic curves are widely used in contemporary architecture. Some famous examples include:

**Sagrada Família by Antoni Gaudí:** Gaudí employed parabolic arches extensively in the design of the church's towers and interior spaces. The parabolas contribute to the building's structural integrity and organic appearance.

**TWA Flight Center by Eero Saarinen:** The flowing, parabolic roof lines of this airport terminal create a futuristic, dynamic form that captures the spirit of flight and travel.

**Heydar Aliyev Center by Zaha Hadid:** This building in Baku, Azerbaijan, uses parabolic curves to create a fluid, undulating design, blending the interior and exterior spaces seamlessly.

### **Parabolic Forms in Islamic Architecture:**

**Muqarnas Vaults:** In traditional Islamic architecture, muqarnas (ornamental vaulting) sometimes employ parabolic elements in the design of ceilings and domes. These curves create a decorative and structurally sound system for covering spaces, often seen in mosques and palaces.

### **Use of parabolic curve in Disaster –resilient\_architecture :**

An explanation of how the elements of architecture that withstand disasters can be modeled mathematically will be debt using the parabolic equation, whereby the higher the building or community resilience the lower the impacts of natural disasters on it.

In this case, a parabolic curve depicts the metamorphosis of disaster resilience over the years as more and more protective measures are adopted. The horizontal axis (x) represents the level or degree of disaster resilience, be it basic or advanced systems, while the vertical axis (y) measures the level of damage that a disaster has inflicted or the impact of the same.

There is a relationship of disaster resilience 'x' and disaster impact 'y' which can be expressed by parabolic curve equation facing downward:

$$y = -ax^2 + bx + c$$

The parabolic curve suggests that at basic disaster resilient architectural design, the negative impact of disasters is greatly lowered at the onset, but later on as more sophisticated measures are taken, the benefits become less. Basic resilient practices, for example, ensure

structural integrity against earthquakes, floods and so on, achieve maximum outcomes in terms of resource savings in the case of disasters occurring. Further allocation of resources for instance high levels of cyclone resistant structures does not correspond losses avert.

### **Use of parabolic curve in Biophilic architecture:**

Biophilic architecture can be related through a concept-parabolic curve-which explains how the incorporation of nature into design will have positive effects on human wellbeing overtime.

Partmently, a parabola describes a relationship between two variables wherein, generally speaking, the rate of change is very slow and is followed by rapid changes that plateau. We can model the impact of the contact with natural elements in biophilic architecture on human wellbeing using a parabolic curve.

Mathematically, the relationship between the degree of biophilic design, x, and human well-being, y, could be described by the quadratic equation representing a parabolic curve:

$$y = ax^2 + bx + c$$

where,

x = degree of biophilic integration

y = human well-being

a, b, c = constants shaping the curve.

From there, as x increases-more biophilic elements-y rockets upward initially before finally leveling out at higher values, showing the positive effects of the biophilic design.

The parabolic curve effectively models the non-linear impact of biophilic architecture on well-being, showing how small initial introductions of nature have minimal effects, rapidly improve as more elements are added, up to an optimal point. This goes to underline the importance of proper integration of natural features within an architectural design.

### **Conclusion**

To sum it up, the elevating conquest concerning the 'Aesthetic Parabolic Curve', rests firmly upon an architectural 'triumph' that is the marriage of structural logic with geometric beauty. Available The extraordinary effectiveness of the parabolic arch in its function as a

structure makes it useful for providing open and spacious environments without many materials or interior columns, due to its spreading of the load over a wider region. The design also has an elegant appealing parabolic form which is more aesthetic to modern designs of buildings. The parabolic curve as an architectural design continues to offer defining and creating innovative forms even in the future as new needs arise. The curved shape integrates both substance and elegance. Thus it is regarded as a modern and precise approach to architectural design.

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