

ANALYSIS OF SUBSTANCE ABUSE CONTROL MODEL IN NIGERIA

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Abstract

In this research, mathematical modelling of treatment control of substance abuse in Nigeria was carried out. We present a deterministic model for the of control treatment of substance abuse in which impatient and outpatient were both important for the disease control. We calculated the basic reproduction number using next generation matrix method and The analytical results reveal that the substance abuse model is mathematically well-posed and defined within the positive region of the parameter space, Λ . The existence of equilibrium states is established, with both the substance abuse-free equilibrium (SAFE) and the substance abuse-endemic equilibrium (SAEE) found to be locally and asymptotically stable under certain conditions, and unstable otherwise. Furthermore, the global stability analysis of the SAEE indicates that it is asymptotically stable. The study also deduces that the control methods employed can effectively reduce the progression rate of substance abuse. Importantly, the analysis highlights that individual involvement and commitment will play a vital role in the successful control of the substance abuse menace in Nigeria. This study provides valuable insights into the mathematical modelling and analysis of substance abuse dynamics, which can

inform the development of effective intervention strategies and policies to address this critical public health challenge in the Nigerian

Keywords: Substance Abuse, Treatment Control, Lyapunov Function, Stability Analysis

INTRODUCTION

Substance abuse, characterized by the harmful and excessive use of drugs, alcohol, and other intoxicating substances, has emerged a persistent public health and security challenge in Nigeria, with far-reaching consequences for individuals, communities, and the nation as a whole. Substance use disorders have been linked to a wide range of adverse outcomes, including physical and mental health problems (Oshodi *et al.*, 2020; Dumbili & Onyima, 2018), criminal activities and violence (Igwe & Ojukwu, 2020; Adekoya *et al.*, 2021), strained social services (Onifade *et al.*, 2021; Amoo & Odejide, 2019), and hindrances to economic development (Okon *et al.*, 2022; Tade & Adekoya, 2021).

In response to this crisis, the Nigerian government has implemented various control and intervention models to curb the production, trafficking, and consumption of illicit substances (Adamson *et al.*, 2019; Bamgbose & Okafor, 2020). However, the dynamics underlying substance abuse in Nigeria remain complex, with evolving patterns of drug use (Oluwole *et al.*, 2020; Aloba *et al.*, 2019), the persistent involvement of organized crime networks (Dambazau & Obuah, 2022; Onyebueke & Ajaero, 2021), and the influence of socioeconomic, cultural, and political factors (Olley & Ogundele, 2020; Ibadin & Onojighofia, 2021).

In a comprehensive nationwide survey, Adebayo and Ogunleye (2021) examined the patterns and determinants of substance abuse among Nigerian youth, finding that the most commonly abused substances included cannabis, alcohol, and prescription opioids. The study, published in the African Journal of Drug and Alcohol Studies, identified risk factors such as peer influence, family dysfunction, and unemployment as key contributors to substance use initiation and dependence. Corroborating these findings, Chukwuma *et al.* (2019), in their report published in the International Journal of Drug Policy, conducted in-depth interviews with healthcare providers and law enforcement officials in Lagos, Nigeria. The authors emphasized the significant strain that substance abuse places on the country's

already overburdened public health and criminal justice systems, underscoring the urgent need for a multifaceted, evidence-based approach to prevention, treatment, and rehabilitation.

Building on these insights, Okafor and Egbonu (2022), in their review article in the Nigerian Medical Journal, highlighted the unique sociocultural and economic factors that contribute to substance abuse in the Nigerian context. The researchers called for the development of culturally-sensitive intervention programs that address the root causes of substance use disorders, such as social marginalization, economic instability, and limited access to mental health services. Complementing these studies, Duru et al. (2020), in their work published in the International Journal of Environmental Research and Public Health, examined the impact of substance abuse on family dynamics and interpersonal relationships in Nigeria. Their findings revealed the profound disruptions and breakdowns in family structures, further exacerbating the cycle of substance use and its associated harms.

Addressing the issue from a public policy perspective, Odejide (2016), in an article in the Journal of Substance Abuse Treatment, advocated for the development of comprehensive national drug control policies in Nigeria, emphasizing the need for a balanced approach that combines demand reduction, harm reduction, and supply control strategies. Highlighting the role of the media, Akande and Akande (2018), in their study published in the Journal of Substance Use, explored the framing of substance abuse narratives in Nigerian news outlets, underscoring the potential for the media to shape public perceptions and influence policy decisions.

Furthermore, Akinola and Akinsola (2017), in their work in the Journal of Psychoactive Drugs, investigated the unique challenges faced by women struggling with substance use disorders in Nigeria, advocating for gender-responsive treatment and support services. Building on these insights, Nwanna and Adesokan (2021), in their article in the International Journal of Mental Health and Addiction, examined the comorbidity of substance abuse and mental health disorders in Nigeria, emphasizing the need for integrated, holistic approaches to addressing these intersecting issues.

MODEL FORMULATION

In this research work, incorporate treatment control of inpatient and outpatient in to the research work of Akpienbi *et al.* (2021), to eradicate substance abuse in Nigeria. The was developed base on the assumption that;

- i. All parameters are positive.
- ii. All susceptible individuals are born or immigrated.
- iii. All individuals in the study have an equal chance of becoming drug users.
- iv. Infected individuals are either treated at or not treated.
- v. Family referrer of drug infected individuals play a major role in treatment.
- vi. Susceptible individuals become drug users through the social contract process only
- vii. We assume drug infected individuals recovered.
- viii. Recovered individuals returned to susceptible class

Model Equation Substance Abuse Control Model

$$\left. \begin{aligned}
 \frac{dS}{dt} &= \pi + \rho\tau R - \beta IS - dS \\
 \frac{dE}{dt} &= \beta IS - \gamma E - dE - \mu E \\
 \frac{dI}{dt} &= \gamma E - (v_1\sigma + v_2\delta + \mu_1 + d)I \\
 \frac{dT_i}{dt} &= v_1\sigma I - (d + \alpha)T_i \\
 \frac{dT_0}{dt} &= v_2\delta I - (d + \eta)T_0 \\
 \frac{dR}{dt} &= \alpha T_1 + \eta T_0 - (d + \rho\tau)R
 \end{aligned} \right\} \tag{1}$$

Table 1 Definition of variable/parameters in drug abuse model

Variable/Parameters	Description
π	Constant birth rate and immigration
β	Interaction between the susceptible and infected individuals

d	Natural Death rate
γ	Substance abuse Infection rate
μ_1	Induced death due to infection
δ	The rate of infected individual in out patient
σ	The rate of infected individual in the inpatient treatment
α	Recovered rate of inpatient
v_1 and v_2	Family roles in referring of infected individual into treatment
μ	Induced death due to exposure to the substance
η	Recovered rate of outpatient
p_r	The proportion of the recovered individual returning into susceptible class

ANALYSIS OF THE MODEL

The substance abuse model analyzes the system without control, establishing the mathematical significance, existence of stationary points, and their stability. The basic reproduction number R_0 and effective reproduction number are computed using the next-generation matrix method. The value of the basic reproduction number R_a is used to define the conditions for global and local stability of the stationary points. The focus is on the dynamic behavior of the substance abuse model with control measures that aim to reduce the effect of substance abuse in Nigeria.

BASIC PROPERTIES OF THE MODEL

Invariant region

Since substance has effect on human population, there are need for model it process. In regard to that this assumption are make defined state variables and parameters used in the model are non-negative for $\Omega \geq 0$. We are required to analyse the drug abuse model so that it is defined in a suitable feasible region which has positive state variables and therefore result to Theorem 4.1; The existing forward solutions in \mathbb{R}_+^6 of the drug abuse model system are feasible $\forall \Omega \geq 0$ whenever they enter the invariant region namely Λ . where

$$\Omega = (S, E, I, T_i, T_0, R) \in (S + E + I + T_i + T_0 + R) : \leq N$$

Since substance abuse has an effect on the human population, there is a need to model its process. In regard to that, the following assumptions are made:

- The state variables and parameters used in the model are non-negative for $\Omega \geq 0$
- The drug abuse model is defined in a suitable feasible region that has positive state variables.

The existing forward solutions in \mathbb{R}_+^6 of the substance abuse model system are feasible $\forall t \geq 0$ whenever they enter the invariant region Λ .

where $\Omega = (S, E, I, T_i, T_0, R) \in (S + E + I + T_i + T_0 + R) : \leq N$

In other words, the substance abuse model is well-defined, and the solutions remain within a positive and bounded region over time.

Positivity of the solution

Here we need to show that variables and parameters used in the drug abuse model are non-negative $\Omega \geq 0$. Assume the initial values of the drug abuse model system (3) to be: $(S(0) > 0, E(0) > 0, I(0) > 0, T_i(0) > 0, T_0(0) > 0, R(0) > 0) \geq 0$. Then the model's solution set $S(t) > 0, E(t) > 0, I(t) > 0, T_i(t) > 0, T_0(t) > 0, R(t) > 0$ are positive $\Omega \geq 0$.

Analysis of the stationary points R_0

This part of the study focuses on the fundamental mathematical properties substance abuse model:

Existence of stationary states: The study establishes the existence of stationary points or equilibrium states in the drug abuse model.

Basic reproduction number (R_0): The study computes the basic reproduction number, which represents the number of new drug use cases produced by one drug user over the entire period of their drug use.

Stability of the stationary points: The study analyses the stability of the stationary points or equilibrium states in the drug abuse model. This includes determining the conditions for both local and global stability of the stationary points.

Substance Abuse Free Equilibrium

At substance abuse free Equilibrium (SBE), we set all the model equation equal to zero.

When $E = I = T_i = T_o = R = 0$

Therefore, the equilibrium point is $E_0 = (S^*, E^*, I^*, T_i^*, T_o^*, R^*) = (\frac{\pi}{d}, 0, 0, 0, 0)$

Basic reproduction number R_0 for substance drug abuse

The basic reproduction number, denoted as R_0 , represents the number of secondary drug abuse cases that are produced by one drug user over the entire period of their drug use in a population defined solely by susceptible individuals.

The criteria for this dimensionless parameter R_0 are as follows:

If $R_0 < 1$, then a single drug user in a population defined by only susceptible individuals may influence less than one individual to start using drugs. This implies that the drug abuse problem may be eradicated from the population, and the drug-free stationary point is asymptotically stable, meaning that the drug abuse cannot persist in the society.

When $R_0 > 1$, it indicates that one drug user in a population defined by only susceptible individuals may influence more than one individual to start using drugs. This means that the drug abuse problem may continue to persist in the society. In this situation, the drug-free equilibrium point is unstable, and it is clear that the drug abuse problem can attack the society and remain for a long time.

If $R_0 = 1$, it suggests that one drug user in an entirely susceptible population may influence drug use to one new individual. Hence, the drug abuse problem will be alive in the country without a serious epidemic.

The basic reproduction number, R_0 , is also known as the spectral radius, which is a critical epidemiological parameter in the drug abuse model.

$$R_0 = \frac{\beta \pi \gamma}{d(V_1 \sigma + V_2 \delta + \mu_1 + d)(\gamma + d)}$$

When early identification and referrer control treatment methods are applied, the number of secondary cases of drug users will decrease. This reduction in the number of drug users

will subsequently decrease the number of adequate contacts between the drug user population and the susceptible population.

Using the same next generation matrix method as before, we can generate an "effective reproduction number" that takes into account the effect of these control measures.

The effective reproduction number represents the number of new drug use cases that are produced by one drug user during their entire infectious period, when the early identification and referrer control treatment interventions are in place.

This effective reproduction number can be used to assess the impact of the control strategies on the dynamics of the drug abuse problem. If the effective reproduction number is less than 1, it suggests that the control measures are effective in limiting the spread of drug abuse in the population. Conversely, if the effective reproduction number is greater than 1, it indicates that the drug abuse problem may still persist despite the implementation of the control interventions.

By analysing the effective reproduction number, researchers and policymakers can evaluate the efficacy of the early identification and referrer control treatment methods in containing and potentially eradicating the drug abuse issue within the population.

$$R_e = \frac{\beta \pi \gamma}{d(\mu_1 + d)(\gamma + d)}$$

When $v_1\sigma, v_2\delta \neq 0$ it reduces the value of the number of secondary cases produced as it can be seen in the expression R_0 it can be seen that the increase of the control efforts will reduce the value of the expression in (R_0) which the number of secondary cases of drug users produced by a drug user in the entire period of drug abuse, but when $v_1\sigma, v_2\delta = 0$ the expression R_0 become the basic reproduction number as given in R_e .

LOCAL STABILITY OF SUBSTANCE ABUSE FREE EQUILIBRIUM

The analysis focuses on establishing the local stability of the drug-free stationary point, E_0 , in the drug abuse model. The approach used is the Jacobian method, which involves analysing the Jacobian matrix of the system evaluated at the drug-free stationary point E_0 .

To prove that the drug-free stationary point E_0 is locally and asymptotically stable, the goal is to show that the real parts of the eigenvalues of the Jacobian matrix at E_0 are negative.

The method used is based on the concept proposed by Akpienbi *et al.* (2021), which states that to ensure the eigenvalues are negative, we need to demonstrate that the determinant of the Jacobian matrix at E_0 is positive and its trace is negative.

By analysing the Jacobian matrix of the system (1) at E_0 , the text leads to the following theorem:

Theorem: The drug-free stationary point E_0 is locally asymptotically stable if and only if the trace of the Jacobian matrix at E_0 is negative, and the determinant of the Jacobian matrix at E_0 is positive.

The paraphrased text then states that the expression for the trace of the Jacobian matrix at E_0 is provided below

$$JE_0 = \begin{pmatrix} -d - \lambda & 0 & -\frac{\beta\pi}{d} & 0 & 0 & \rho\tau \\ 0 & -\gamma - d - \mu - \lambda & \frac{\beta\pi}{d} & 0 & 0 & 0 \\ 0 & \gamma & -v_1\sigma - v_2\delta - \mu_1 - d - \lambda & 0 & 0 & 0 \\ 0 & 0 & 0v_1\sigma & -d - \alpha - \lambda & 0 & 0 \\ 0 & 0 & v_2\delta & 0 & -d - \eta - \lambda & 0 \\ 0 & 0 & 0 & \alpha & \eta & -d - \rho\tau - \lambda \end{pmatrix}$$

$$\text{trace} = -d - (\gamma + d + \mu) - (v_1\sigma + v_2\delta + \mu_1 + d) - (d + \alpha) - (d + \eta) - (d + \rho\tau)$$

Therefore the matrix of E_0 is negative. We then compute determinant of the matrix E_0 in which it is positive if $\frac{\beta\pi\gamma}{d(\gamma+\gamma)(v_1\sigma+v_2\delta+\mu_1+d)} < 1$

where $\frac{\beta\pi\gamma}{d(\gamma+\gamma)(v_1\sigma+v_2\delta+\mu_1+d)}$ represents the basic reproduction number,

The above results justifies that the substance abuse free stationary point E_0 is locally asymptotically stable as in theorem below: The Drug Free stationary point E_0 is locally asymptotically stable if $R < 1$ and unstable if $R > 1$.

GLOBAL STABILITY ANALYSIS OF SUBSTANCE ABUSE ENDEMIC EQUILIBRIUM POINT

This section analyzes the conditions for the stability of the drug abuse endemic equilibrium points. Building on the results from the study by Rhodes (2011), the analysis shows that the

local stability of the Substance Abuse Free Equilibrium (SAFE) implies the local stability of the Substance Abuse Endemic Equilibrium (SAEE) under the reverse condition.

To further investigate the global stability of the SAEE, the approach taken is based on the work by Korobeinikov (2013 & 2007). Specifically, the analysis involves formulating a suitable Lyapunov function for the drug abuse model system.

- Establish the local stability of the SAFE, which then implies the local stability of the SAEE under the reverse condition.
- Utilize the Korobeinikov approach to derive a Lyapunov function that can be used to analyze the global stability of the SAEE.

By constructing the appropriate Lyapunov function for the drug abuse model, the analysis aims to determine the global stability conditions for the drug abuse endemic equilibrium point as

$$V = \sum a_i (y_i - y_i^* \ln y_i)$$

where a_i and y_i is an appropriately selected positive constant, y_i is a population of the i^{th} class type, and y_i^* is the stationary point then it becomes

$$\text{then, } V = A_1 (S - S^* \ln S) + A_2 (E - E^* \ln E) + A_3 (I - I^* \ln I) + A_4 (T_i - T_i^* \ln T_i) \\ + A_5 (T_0 - T_0^* \ln T_0) + A_6 (R - R^* \ln R)$$

The constants are non-negative in Λ such that $A_i > 0$ for $i = 1, 2, 3, \dots, 6$. Λ which is the Lyapunov function $A_1, A_2, A_3, \dots, A_6$ defined as the Lyapunov function constant are selected so that Λ should be continuous and differentiable in a space.

Finding the time derivative of V to have

$$\frac{dV}{dt} = A_1 \left(1 - \frac{S^*}{S}\right) [\pi + \rho\tau R - \beta IS - dS] + A_2 \left(1 - \frac{E^*}{E}\right) [\beta IS - (d + \gamma + \mu)E] \\ + A_3 \left(1 - \frac{I^*}{I}\right) [\gamma E - (v_1\sigma + v_2\delta + \mu_1 + d)I] + A_4 \left(1 - \frac{T_i^*}{T_i}\right) [v_1\sigma I - (d + \alpha)T_i] \\ + A_5 \left(1 - \frac{T_0^*}{T_0}\right) [v_2\delta I - (d + \eta)T_0] + A_6 \left(1 - \frac{R^*}{R}\right) [\alpha T_i + \eta T_0 - (\rho\tau + d)R]$$

The time derivative computing of V at substance abuse endemic stationary point yields

$$\frac{dV}{dt} = -A_1 \left(1 - \frac{S^*}{S}\right) - A_2 \left(1 - \frac{E^*}{E}\right) - A_3 \left(1 - \frac{I^*}{I}\right) - A_4 \left(1 - \frac{T_i^*}{T_i}\right) - A_5 \left(1 - \frac{T_0^*}{T_0}\right) \\ - A_6 \left(1 - \frac{R^*}{R}\right) + F(S, E, I, T_i, T_0, R)$$

The function $F(S, E, I, T_i, T_0, R)$ is negative

Applying research work of Matowo (2013) and Korobeinikov (2004)

then $F(S, E, I, T_i, T_0, R) \leq 0$ for all S, E, I, T_i, T_0, R

thus, $\frac{dV}{dt} \leq 0$ for all S, E, I, T_i, T_0, R

and it zero whenever, $S = S^*, E = E^*, I = I^*, T_i = T_i^*, T_0 = T_0^*, R = R^*$

Therefore, the largest compact invariant set in S, E, I, T_i, T_0, R such that $\frac{dV}{dt} = 0$ is the singleton E_1 which is substance abuse endemic stationary point of the model system (1).

By LaSalle's invariant principle in Korobeinikov (2007) the model proclaims that E_1 is globally asymptotically stable in the interior of substance abuse model system region of S, E, I, T_i, T_0, R . If $R > 1$ then substance abuse model system (1) has a distinctive substance abuse endemic equilibrium point E_1 which is globally asymptotically stable in S, E, I, T_i, T_0, R .

CONCLUSION

The dynamics model of human population when substance abuse is a paramount community in Nigeria was considered. The dynamical behaviour of the model population is depicted to when treatment control measure of inpatient and outpatient effort is applied to eliminate or reduce the rate of substance abuse. Furthermore, the analytical result revealed that substance abuse model is mathematically well posed and it defines in the positive region of Λ . The existence of the equilibrium state condition was established for substance abuse free and endemic equilibrium points are both found to be locally and asymptotically stable when $R_0 < 1$ and unstable otherwise. Also, the global stability analysis of the endemic equilibrium of substance abuse indicates to be asymptotically stable.

However, the control method deduces the progression rate of substance abuser. Now, it is beyond doubt that individual involvement will play a vital role for the successful control of substance abuse menace in Nigeria.

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