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# ANALYTICAL ANALYSIS OF COMMON FIXED POINT RESULTS IN FUZZY CONE METRIC SPACES

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**Abstract**

The concept of fuzzy sets was introduced by Zadeh (1965) which marked the beginning of the evolution of fuzzy mathematics. The introduction of uncertainty in the theory of sets in a non-probabilistic manner opened up new possibilities for research in this field. Since then, many authors have explored the theory of fuzzy sets and its applications, leading to successful advancements in various fields such as mathematical programming, model theory, engineering sciences, image processing, and control theory. In this paper, we aim to improve and generalize some common fixed point theorems in fuzzy cone metric spaces, which is an extension of the well-known results given by Saif Ur Rahman and Hong Xu-Li (2s017).

**Keywords**: Fuzzy metric space, Fuzzy Cone Metric Space, Fuzzy Cone Contractive Mapping

## INTRODUCTION

In 2007, Huang and Zhang proposed the concept of cone metric spaces, which involves substituting a real number with a Banach space. They also established some fixed point theorems for contractive mapping. Since then, numerous studies have focused on the issues related to cone metric spaces.

The fuzzy set theory was introduced by Zadeh (1965) there has been a great effort to obtain fuzzy analogues of classical theories. In 1970, Kramosil and Michalek, introduced the fuzzy metric spaces. Later on, George and Veeramani (1994) gave a stronger form of metric fuzziness. Some fixed point results for set valued mappings on fuzzy metric spaces were found in [M. Grabiec(1988), Sadeghi (1960), Hadzic and Pap, E. (2002), and Kiani and Harad(2011)].

In 2015, Oner et.al. Introduced the notation of fuzzy cone metric space, which is a generalization of the notation of fuzzy metric space by George and Veeramani (1994). They also presented some structural properties of fuzzy cone metric spaces and proved a fixed point theorem under a fuzzy cone contractive condition. Some fixed point theorems and common fixed point theorems concerning fuzzy cone metric spaces were obtained by Ali & kanna (2017) & Priyobatra et al. (2016) also obtained some more properties in fuzzy cone metric spaces of results ([Oner, T, (2016)]).

In this article, we generalize and improve the results of Saif Ur, Rehman (2017) and obtain common fixed point theorem in fuzzy cone metric space for contractive condition.

## PRELIMINARY NOTES

**Definition 1 ([Schweizer, A. and Sklar, (2020)]):** A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if it satisfies the following conditions:

(1) $*$ is commutative and associative

(2) $*$ is continuous

(3) $\boldsymbol{1 * a = a}$ for all $\boldsymbol{a \in [0,1]}$

(4) $\boldsymbol{a * b \leq c * d}$, whenever $\boldsymbol{a \leq c}$ and $\boldsymbol{b \leq d}$, for each $\boldsymbol{a, b, c, d \in [0,1]}$.

**Definition 2 ([George and Veeramani (1994) 4]):** A fuzzy metric space is an ordered triple $(X, M,*)$ such that $X$ is a non- empty set, $*$ is continuous t-norm and $M$ is a fuzzy set on $X \times X \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X, s, t > 0$

$(i)$ $M(x, y, t) > 0$

$(ii)$ $M(x, y, t) = 1$ iff $x = y$

$(iii)$ $M(x, y, t) = M(y, x, t)$

$(iv)$ $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

$(v)$ $M(x, y, .): (0, \infty) \rightarrow [0,1]$ is continuous.

**Definition 3 ([Huang and Zhang(2007)]):** A subset P of E is called a cone if

(a) P is closed, nonempty, and $P \neq \{\theta\}$

(b) If $a, b \in [0, \infty]$ and $x, y \in P$, then $ax + by \in P$

(c) If both $x \in P$ and $-x \in P$, then $= \theta$ .

For a given cone $P \subset E$, a partial ordering $\leq$ on E via $P$ is defined by $x \leq y$ if and only if $y - x \in P. x < y$ stands for $x \leq y$ and$x \neq y$, while $x \ll y$ stand for $y - x \in int(p)$.

**Definition 4 ([Oner, T .et al. (2015)]):** A fuzzy cone metric space is a 3-tuple $(X, M,*)$ such that $P$ is a cone of$E$,$X$ is a nonempty set, $*$ is a continuous t-norm and $M$ is a fuzzy set on $X^2 \times int(P)$. Satisfying the following conditions, for all $x, y, z \in X$ and $t, s \in int(P)$ $(that\ is\ t \gg 0, s \gg 0)$

1. $M(x, y, t) > 0$
2. $M(x, y, t) = 1\ iff\ x = y$
3. $M(x, y, t) = M(y, x, t)$
4. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$
5. $M(x, y, .): int(p) \rightarrow [0,1]$ is continuous.

**Definition 5 ([Oner, T .et al. (2015)]):** Let $(X, M,*)$ be a fuzzy cone metric space, $x \in X$ and $\{x_n\}$ be a sequence in $X$. Then,

(1) $\{x_n\}$ is said to converge to $x$ if for $t \gg \vartheta$ and $\propto \in (0,1)$, there exist a natural number $n_1$such that $M(x_n, x, t) > 1 - \propto$ for all$n > n_1$. It is denoted as

$$\lim_{n \to \infty} x_n = x;$$

(2) $\{x_n\}$ is said to Cauchy sequence if for $\propto\in(0,1)$ and $t\gg\vartheta$, there exists natural number $n_1\in N$ such that $M(x_n,x_m,t)>1-\propto$ for all $n,m>n_1$;

(3) $(X,M,*)$ is said to be complete fuzzy cone metric space if every Cauchy sequence is convergent in $X$;

(4) $\{x_n\}$ is said to be fuzzy cone contractive if there exists $\propto\in(0,1)$ such that

$$\frac{1}{M(x_{n+1},x_{n+2},t)}-1\leq\propto\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right) for\ all\ t\gg\vartheta, n\in N.$$

**Definition 6 ([Saif Ur Rehman and Hong-Xu Li. (2017),]).** Let $(X,M,*)$ be a fuzzy cone metric space. The fuzzy cone metric M is triangular if

$$\frac{1}{M(x,z,t)}-1\ \leq\ \left(\frac{1}{M(x,y,t)}-1\right)+\left(\frac{1}{M(y,z,t)}-1\right)$$

For all $x,y,z\in X$ and each $t\gg\theta$.

**Theorem 7 [Saif Ur Rehman and Hong-Xu Li. (2017)]:** Let $(X,M,*)$ be a complete fuzzy cone metric space in which fuzzy cone contractive sequence are Cauchy and $T:X\to X$ be a fuzzy cone contractive mapping. Satisfies

$$\frac{1}{M(T_x,T_y,t)}-1\leq a\left(\frac{1}{M(x,y,t)}-1\right)+b\left(\frac{1}{M(x,T_x,t)}-1\right)$$
$$+c\left(\frac{1}{M(y,T_y,t)}-1\right)+d\left(\frac{1}{M(y,T_x,t)}-1\right) \qquad (7.1)$$

And symmetrically

$$\frac{1}{M(T_x,T_y,t)}-1\leq a\left(\frac{1}{M(x,y,t)}-1\right)+b\left(\frac{1}{M(x,T_x,t)}-1\right)$$
$$+c\left(\frac{1}{M(y,T_y,t)}-1\right)+d\left(\frac{1}{M(x,T_y,t)}-1\right) \qquad (7.2)$$

For each $x,y\in X$ and $t\gg\theta$ . Then T has a unique fixed point.

## MAIN_RESULTS

In this section we have extend and generalize the theorem 7 and obtain common fixed point result in fuzzy cone metric space. The main results are as follows:

**Theorem 8.** Let $(X,M,*)$ be a complete fuzzy cone metric space in which fuzzy cone contractive sequence are Cauchy and $M$ is triangular and $T_1,T_2:X\to X$ be a fuzzy cone contractive mapping. Then, $T_1$ and $T_2$ have a unique common fixed point, under the following fuzzy cone contractive type conditions:

$$\frac{1}{M(T_1x,T_2y,t)} - 1 \le a\left(\frac{1}{M(x,y,t)} - 1\right) + b\left(\frac{1}{M(x,T_1x,t)} - 1\right)$$
$$+c\left(\frac{1}{M(y,T_2y,t)} - 1\right) + d\left(\frac{1}{M(y,T_1x,t)} - 1\right) \quad (8.1)$$

and symmetrically

$$\frac{1}{M(T_1x,T_2y,t)} - 1 \le a\left(\frac{1}{M(x,y,t)} - 1\right) + b\left(\frac{1}{M(x,T_1x,t)} - 1\right)$$
$$+c\left(\frac{1}{M(y,T_2y,t)} - 1\right) + d\left(\frac{1}{M(x,T_2y,t)} - 1\right) \quad (8.2)$$

Where $t \gg \theta, a, b, c, d \in [0, \infty]$ with $a + b + c < 1$.

**Proof :** Let $x_0 \in X$ and $\{x_{2n}\}$ and $\{x_{2n+1}\}$ be any two square in X, such that

$$x_{2n} = T_1x_{2n-1} = T_1^{2n}\, x_0$$

and

$$x_{2n+1} = T_2x_{2n} = {T_2}^{2n+1}x_0$$

Then by equation (8.1)for $t \gg \theta, n \gg 1$.

$$\frac{1}{M(x_{2n},x_{2n+1},t)} - 1 = \frac{1}{M(T_1x_{2n-1},T_2x_{2n},t)} - 1$$
$$\le a\left(\frac{1}{M(x_{2n-1}, x_{2n}, t)} - 1\right) + b\left(\frac{1}{M(x_{2n-1}, T_1x_{2n-1}, t)} - 1\right)$$
$$+ c\left(\frac{1}{M(x_{2n},T_2x_{2n},t)} - 1\right) + d\left(\frac{1}{M(x_{2n},T_1x_{2n-1},t)} - 1\right)$$
$$= a\left(\frac{1}{M(x_{2n-1},x_{2n},t)} - 1\right) + b\left(\frac{1}{M(x_{2n-1},x_{2n},t)} - 1\right)$$
$$+ c\left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) + d\left(\frac{1}{M(x_{2n},x_{2n},t)} - 1\right)$$

$$(1-c)\left(\frac{1}{\mathrm{M}(x_{2n},x_{2n+1},t)} - 1\right) = a + b\left(\frac{1}{\mathrm{M}(x_{2n-1},x_{2n},t)} - 1\right)$$

Implies that

$$\left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) = \frac{a+b}{(1-c)}\left(\frac{1}{M(x_{2n-1},x_{2n},t)} - 1\right)$$

Thus $$\left(\frac{1}{M(x_{2n},x_{2n+1},t)} - 1\right) \le k\left(\frac{1}{M(x_{2n-1},x_{2n},t)} - 1\right)$$

$$\text{Where } k = \frac{(a+b)}{(1-c)} \leq 1 \text{ since } a+b+c<1$$

Implies that $$\frac{1}{M(x_{2n}, x_{2n+1}, t)} - 1 \leq k\left(\frac{1}{M(x_{2n-1}, x_{2n}, t)} - 1\right)$$

$$\leq \cdots \ldots \ldots \leq k^n\left(\frac{1}{M(x_0, x_1, t)} - 1\right)$$

Which means that $\{x_{2n}\}$ is a fuzzy cone contractive sequence, and we get

$$\lim_{n\to\infty} M(x_{2n}, x_{2n+1}, t) = 1 \text{ for } t \gg \theta \tag{8.3}$$

Noticing that M is triangular, then for all $m > n \geq n_0$

$$\frac{1}{(x_{2n}, x_{2m}, t)} - 1 \leq \left(\frac{1}{M(x_{2n}, x_{2n+1}, t)} - 1\right) + \left(\frac{1}{M(x_{2n+1}, x_{2n+2}, t)} - 1\right)$$

$$+ \ldots\ldots + \left(\frac{1}{M(x_{2n-1}, x_{2n}\, t)} - 1\right)$$

$$\leq k^n\left(\frac{1}{M(x_0, x_1, t)} - 1\right) + k^{n+1}\left(\frac{1}{M(x_0, x_1, t)} - 1\right)$$

$$+ \ldots\ldots + .k^{m-1}\left(\frac{1}{M(x_0, x_1, t)} - 1\right)$$

$$\leq (k^n + k^{n+1} + \cdots \ldots \ldots + k^{m-1})\left(\frac{1}{M(x_0\, x_1, t)} - 1\right)$$

$$= \frac{k^n}{1-k}\left(\frac{1}{M(x_0, x_1, t)} - 1\right) \to 0 \text{ as } n \to \infty.$$

What yields that $\{x_{2n}\}$ is a Cauchy sequence in X since $(X, M, *)$ is complete.

There exists $w \in X$ such that

$$\lim_{n\to\infty} M(x_{2n}, w, t) = 1 \text{ for } t \gg \theta$$

Since $M$ is triangular, we have

$$\frac{1}{M(w, T_1 w, t)} - 1 \leq \left(\frac{1}{M(w, x_{2n}, t)} - 1\right) + \left(\frac{1}{M(x_{2n}, T_1 w, t)} - 1\right) \text{ for } t \gg \theta \tag{8.4}$$

Now,

$$\frac{1}{M(x_{2n}, T_1 w, t)} - 1 = \left(\frac{1}{M(x_{2n}, T_1\, x_{2n-1}, t)} - 1\right) + \left(\frac{1}{M(T_1 X_{2n-1}, T_1 w, t)} - 1\right)$$

$$= \left(\frac{1}{M(x_{2n}, x_{2n}, t)} - 1\right) + \left(\frac{1}{M(T_1 x_{2n-1} T_1 w, t)} - 1\right)$$

$$= \left(\frac{1}{M(T_1 x_{2n-1} T_1 w, t)} - 1\right)$$

$$\leq a\left(\frac{1}{M(x_{2n-1}, w, t)} - 1\right) + b\left(\frac{1}{M(x_{2n-1}, T_1 x_{2n-1}, t)} - 1\right)$$

$$+ c\left(\frac{1}{M(w, T_1 w, t)} - 1\right) + d\left(\frac{1}{M(w, T_1 x_{2n-1}, t)} - 1\right) \to 0$$

as $n \to \infty$

This implies that

$$\lim_{n\to\infty} sup\left(\frac{1}{M(x_{2n}, T_1 w, t)} - 1\right) - 1 \leq c\left(\frac{1}{M(w, T_1 w, t)} - 1\right) \text{ for } t \gg \theta \qquad (8.5)$$

Together with equation (8.4) & (8.5), we get

$$\Rightarrow \frac{1}{M(w, T_1 w, t)} - 1 \leq c\left(\frac{1}{M(w, T_1 w, t)} - 1\right) \text{ for } t \gg \theta$$

Since that $c < 1$ and $a + b + c < 1$. So, $M(w, T_1 w, t) = 1$. That is $T_1 w = w$

Hence $w$ is a fixed point of $T_1$.

Similarly we can prove that $T_2 w = w$. Therefore, $T_1 w = w = T_2 w$.

So, $w$ is common fixed point of $T_1$ and $T_2$.

Now, if $\mu$ is another common fixed point of $T_1$ and $T_2$. Then

$$\frac{1}{M(T_1 w, T_2 v, t)} - 1 \leq a\left(\frac{1}{M(w, v, t)} - 1\right) + b\left(\frac{1}{M(w, T_1 w, t)} - 1\right)$$

$$+ c\left(\frac{1}{M(v, T_2 v, t)} - 1\right) + d\left(\frac{1}{M(v, T_1 w, t)} - 1\right)$$

$$= a\left(\frac{1}{M(w, v, t)} - 1\right) + b\left(\frac{1}{M(w, w, t)} - 1\right)$$

$$+ c\left(\frac{1}{M(v, v, t)} - 1\right) + d\left(\frac{1}{M(v, w, t)} - 1\right)$$

$$= a + d\left(\frac{1}{M(v, w, t)} - 1\right)$$

$$\leq a + d\left(\frac{1}{M(v, w, t)} - 1\right)$$

Since $a + d < 1$ . So, we get $M(v, w, t) = 1$. that is $v = w$.

Thus $w$ is the unique common fixed point of $T_1$ and $T_2$ .

These completed the proof of the Theorem.

**Corollary 9:** Let $(X, M,*)$ be a complete fuzzy cone metric space in which fuzzy cone contractive sequence are Cauchy and $M$ is triangular and $T: X \rightarrow X$ be a fuzzy cone contractive mapping. Then, $T_1$ and $T_2$ have a unique common fixed point, under the following fuzzy cone contractive type conditions:

$$\frac{1}{\mathrm{M}(T_1\mathrm{x}, T_2\mathrm{y}, \mathrm{t})} - 1 \leq \mathrm{a}\left(\frac{1}{\mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t})} - 1\right)$$

**Proof: -** If we take $b = c == d = 0$. Then we get the required corollary.

**Corollary 10:** Assume that $(X, M,*)$ is a complete fuzzy cone metric space in which fuzzy cone contractive sequence are Cauchy and $T: X \rightarrow X$ satisfies $b = c = d$ in (8.1) then $T$ has a unique common fixed point in $X$.

**Proof:** $\frac{1}{\mathrm{M(Tx,Ty,t)}} - 1 \leq \mathrm{a}\left(\frac{1}{\mathrm{M(x,y,t)}} - 1\right) + \mathrm{b}\left(\frac{1}{\mathrm{M}(x, T_x, t)} - 1\right)$

$$+\mathrm{b}\left(\frac{1}{\mathrm{M}(y, Ty, \mathrm{t})} - 1\right) + \mathrm{b}\left(\frac{1}{\mathrm{M(y, Tx, t)}} - 1\right)$$

$$\frac{1}{\mathrm{M(Tx,Ty,t)}} - 1 \leq \mathrm{a}\left(\frac{1}{\mathrm{M(x,y,t)}} - 1\right)$$

$$+b\left[\left(\frac{1}{\mathrm{M}(x, T_x, t)} - 1\right) + \left(\frac{1}{\mathrm{M}(y, Ty, \mathrm{t})} - 1\right) + \left(\frac{1}{\mathrm{M(y,Tx,t)}} - 1\right)\right]$$

$$\frac{1}{\mathrm{M(Tx,Ty,t)}} - 1 \leq \mathrm{a}\left(\frac{1}{\mathrm{M(x,y,t)}} - 1\right)$$

$$+b\left[\left(\frac{1}{\mathrm{M}(x,x,t)} - 1\right) + \left(\frac{1}{\mathrm{M}(y,\mathrm{y},\mathrm{t})} - 1\right) + \left(\frac{1}{\mathrm{M(y,x,t)}} - 1\right)\right]$$

$$\frac{1}{\mathrm{M(Tx,Ty,t)}} - 1 \leq \mathrm{a}\left(\frac{1}{\mathrm{M(x,y,t)}} - 1\right) + b\left[\left(\frac{1}{\mathrm{M(y,x,t)}} - 1\right)\right]$$

$$\frac{1}{\mathrm{M(Tx,Ty,t)}} - 1 \leq \mathrm{a} + b\left(\frac{1}{\mathrm{M(x,y,t)}} - 1\right).$$

Since $\mathrm{a} + b < 1$. So, we get $M(x, y, t) = 1$. That is $x = y$

Thus $T$ has unique common fixed point of $X$.

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