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PROPERTIES OF INFINITE MATRICES AND SEQUENCE SPACES

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Abstract

Some general theorems on the product of matrices and their applications to infinite series have been obtained. All theorems include, as special cases, a set of well-known results. Several specific new results can also be deduced from it.

Keywords: Infinite matrices, T-matrix, α -matrix and γ -matrix

Introduction

The theory of summability transformations provides a means of assigning limits to divergent sequences by generalizing the concept of the limit of a sequence. These transformations can be categorized in the following ways: 1) Sequence-to-sequence transformations, and 2) Sequence-to-function transformations.

To create regular transformations, we may utilize the Silverman-Toeplitz theorem, which provides the necessary and sufficient criteria for a matrix to represent a regular method.

Several researchers have studied the distinctive features of sequence to a sequence, series to sequence, and series to series transformations matrices; for a thorough description, (see Dienes,1950) and (Cooke,1931 and 2022). The Silverman-Toeplitz theorem gives us the necessary and sufficient conditions for a matrix to represent a standard method, a method we can then use to generate regular transformations. Dienes (1950) and Cooke (1931 and 2022) and others have studied the unique properties between sequence to sequence, series to sequence, and series to series transformations matrices. Hill has defined the sequence-to-series transition in 1939. Vermes (1946, 1949, 19551) and Ramanujan (1956) have investigated the products of two of the aforementioned matrices. A sequence can be transformed by a T-matrix, and specific examples include the methods of Cesàro, Riesz, Borel, Lindelöf, and Mittage-Leffler (Kirisci, 2015). Ray (2014), Kirisci (2015 and 2016), Nizamuddin (2021), Gangle and Sheikh (2012 and 2014), Altay and Basar (2007,2005 and 2003) and Sahani and Sahani et al. (2021 and 2022)were great mathematician who established some sequence spaces and their matrix transformation, integrated and differential sequence spaces, on the Taylor sequence space of non-absolute type which include the different spaces, the application domain of infinite matrices on classical sequence spaces, some applications of infinite matrices on some spaces, infinite matrices and almost bounded sequences, a note on almost convergence and some matrix transformations, the application of infinite matrices with algorithm, certain topological properties and the duals of the domain of a triangle matrix in sequence spaces on some Euler sequence spaces of non-absolute type, on some spaces of p-bounded variation and related matrix mapping, a certain studies on degree of approximation of functions by matrix transformation, and on certain series to series transformation and analytic continuations by matrix methods respectively.

Definition 1: An infinite matrix

$P = (p_{\ell,g})$ is a γ -matrix if it satisfies the following conditions:

$$\sum_{g=1}^{\infty}\left|p_{\ell,g} - p_{\ell,g+1}\right| \leq M \forall\, \ell \geq 1 \quad (1)$$

and $$p_{\ell,g} \to 1 \text{ as } \ell \to \infty \forall g \quad (2)$$

Definition 2: Let $Q = (q_{\ell,g})$ be an infinite matrix. An infinite matrix Q is said to be a T-matrix if it satisfies the following conditions:

$\sum_{g=1}^{\infty} |q_{\ell,g}| \leq N \forall \ell \geq 1,$

(3)

$\sum_{g=1}^{\infty} q_{\ell,g} \rightarrow 1$ as $\ell \rightarrow \infty,$

(4)

and $q_{\ell,g} \rightarrow 0$ as $\ell \rightarrow \infty \forall g$ (5)

The transformation of the series $\sum a_g$ by the matrix $P \equiv (p_{\ell,g})$ into a convergent series $\sum a_\ell$, so that $b_\ell = \sum_g p_{\ell g} a_g$.

It is abundantly evident that the sufficient and necessary condition for the matrix P to be regular is that,, in which case we shall call it an α -matrix and the product of a γ -matrix and an α -matrix is a γ -matrix, and the product of two α -matrices is an α -matrix (Vermes, 1949).

The matrix $P \equiv (p_{\ell g})$ is an α -matrix if and only if $Q = q_{0g} + q_{1g} + \ldots + q_{\ell g}$ is a γ -matrix.

α -matrix satisfies the following conditions:

$\sum_{g=0}^{\infty} |q_{\ell g} - q_{\ell g+1}| \leq M$ for $\ell = 0,1,2, \ldots$

and $\sum_{\ell=0}^{\infty} q_{\ell g} = 1$ for every g (Vermes, 1949).

Let P is the lower semi-γmatrix of ordinary convergence

$$P \equiv \begin{matrix} 1,0,0,\ldots \\ 1,1,0,\ldots \\ 1,1,1,\ldots \\ -,-,-,\ldots \end{matrix}$$

then the unit matrix is the corresponding α -matrix.

Note (1): $P(\lambda)$ is an α -matrix iff $0 \leq \lambda \leq 1$.

(2) $P(\lambda)$ is an α -matrix iff it is non-negative (Vermes, 1949).

(3) the product αT may not exist.

Proof: Let Q be C (1) matrix (see Cooke, (1950)) and

$$P = \begin{bmatrix} 1 & 1 & 1 & 1\ldots \\ 0 & 0 & 0 & 0\ldots \\ 0 & 0 & 0 & 0\ldots \\ \ldots & \ldots & \ldots & \ldots\ldots \end{bmatrix}$$

then P is α, Q is T, and PQ does not exist.

Main theorems:

Let $P \equiv (p_{\ell g})$ be a γ_A matrix. Let, in addition

$$\lim_{g\to\infty} p_{\ell g} = p_\ell \tag{6}$$

where p_ℓ exists finitely.

We prove the following theorems:

Theorem 1: The row limit of α -matrix corresponding to γ -matrix exists and is finite.

Theorem 2: The row limit of the product R=PQ of a γ -matrix P and an α -matrix Q exists and is

$$r_\ell = \sum_{i=1}^{\infty} p_{\ell i} q_i \tag{7}$$

Theorem 3: The elements of a γ -matrix are bounded.

Theorem 4: The row limit of the product D=QE of two matrices exists and is

$$d_\ell = \sum_{i=1}^{\infty} q_{\ell i} \cdot e_i \tag{8}$$

Theorem 5: If P is a γ -matrix, the matrix defined by $\left(\frac{1}{p_{\ell,g}}\right)$ is also a γ -matrix iff $\frac{1}{p_{\ell,g}} \geq L > 0$ where

$L = \sum_{g=0}^{\infty} |\lambda_g|$ is finite and $\sum_{g=0}^{\infty} \lambda_i = \lambda \neq 0$.

Proof of theorem 5: for sufficient condition,

$$\sum_{g=1}^{\infty} \left| \frac{1}{p_{\ell,g}} - \frac{1}{p_{\ell,g+1}} \right| =$$

$\sum_{g=1}^{\infty}\left|\frac{p_{\ell,g+1}-p_{\ell,g}}{p_{\ell,g}.p_{\ell,g+1}}\right| \leq \frac{M}{L^2}$ (using theorem 3 and given condition)

and $\frac{1}{p_{\ell,g}} \to 1$ as $\ell \to \infty$

The necessity follows from (Sahani, 2022).

Definition 3: The average value of the series as a whole $\sum u_g$ by the matrix P is

$$t = \lim_{\ell\to\infty} \sum_{g=1}^{\infty} p_{\ell,g} u_g \tag{9}$$

assuming that the infinite series on the right hand side is convergent for all l, and that there is a limit to the sum of its terms, which can be written as $l \to \infty$.

Definition 4: The matrix P is regular if $\lim_{\ell\to\infty} \sum_{g=1}^{\infty} p_{\ell,g} u_g \rightleftarrows \lim_{\ell\to\infty} \sum_{g=1}^{\infty} p_{\ell,g+1} u_g$ (10)

Definition 5: The matrix P is semi-regular if $\lim_{\ell\to\infty} \sum_{g=1}^{\infty} p_{\ell,g} u_g \overrightarrow{\rightarrow} \lim_{\ell\to\infty} \sum_{g=1}^{\infty} p_{\ell,g+1} u_g$ (11)

Definition 6: Let $S = (s_{\ell,g})$ be an infinite matrix. If we remove the first r columns of S, then we obtain a new matrix $S^g = (S_{\ell,g+r})$ then it is known as r^{th} diminutive of S.

Theorem 6: The diminutive of a γ -matrix P is a γ -matrix. If P is regular or semi-regular then it becomes $P^{(r)}$.

Proof of theorem 6: It is clear that the equations (1) and (2) are satisfied by $P^{(r)}$.

Also, regularity or semi-regularity follows from the following identity

$$\sum_{g=1}^{\infty} p_{\ell,g+r} u_g = \sum_{g=1+r}^{\infty} p_{\ell,g} u_{g-r}$$

and from definition 4 or definition 5.

Definition 7: Let $S = (s_{\ell,g})$ be an infinite matrix. The matrix S^{rx}, is obtained from the matrix S by repeating each column r^{th} times. Then it is known as r fold sketched matrix.

Theorem 7: A sketched γ -matrix is a γ -matrix. Sketching may destroy regularity or semi-regularity.

Proof of theorem 7: If the matrix S satisfies the definition (1), then we can easily obtain S^{rx}.

For next part,

we consider Borel's γ -matrix, $p_{\ell,g} = \frac{1}{g!}\int_0^{\ell} e^{-z}.z^g dz$

$$= 1 - e^{-\ell}\left(1 + \frac{\ell}{1!} + \frac{\ell^2}{2!} + \ldots + \frac{\ell^g}{g!}\right)$$

which sums the series $1 - 1 + 1 - 1 + \ldots$ (where ℓ and g is non-negative number) to the value $\frac{1}{2}$.

By def^n of P^{2x}, we obtain

$$t_\ell = \sum_{g=1}^{\infty} p_{\ell,g}^{2x}(-1)^g$$

$$= p_{\ell,0} - p_{\ell,0} + p_{\ell,1} - p_{\ell,1} + \ldots = 0$$

and $t_\ell^1 = \sum_{g=1}^{\infty} p_{\ell,g+1}^{2x}(-1)^g$

$= p_{\ell,0} - p_{\ell,1} + p_{\ell,1} - p_{\ell,2} + p_{\ell,2} \ldots = p_{\ell,0}$.

$\therefore t_\ell \to 0, t_\ell^1 \to 1$, shows that $P^{2x}$is not semi-regular, while P is semi-regular. It is clear that P^{2x} is not consistent with P.

Theorem 8: Let $z_g = x_g + iy_g$ lie in an angle $\psi(\psi < \pi)$ of the complex plane whose vertex at the origin and let $\lim_{g\to\infty}\left|\sum_{\ell=1}^{g} z_\ell\right| = \infty$. If the non-negative γ -matrix $\left(P_g(w)\right)$ transforms $\sum_{g=1}^{\infty} z_g$ into $\gamma(w)$, then $\lim_{w\to\infty}|\gamma(w)| = \infty$ (where $g > g_0$).

Proof of theorem 8: Case 1: Let $z_g$be non-negative real numbers for $g > g_0$, then for a number $r > g_0$,

$$\sum_{g=1}^{\infty} P_g(w).z_g = \left[\sum_{g=1}^{r} + \sum_{g=r+1}^{\infty}\right] p_r(w)z_g$$

$$= \sum_{g=1}^{r} p_r(w)z_g + \sum_{g=r+1}^{\infty} p_r(w)z_g$$

≥ 0 and the limit of the first sum on the right is

$\sum_{g=1}^{r} z_g = S_r$, where S_r is the r^{th} partial sum of the infinite series $\sum_{g=}^{\infty} z_g$.

Thus all the points of left hand side are $\geq S_r$i.e. r^{th} partial sum.

$\therefore \quad \lim_{g\to\infty} |S_g| = \infty$

i.e. $\quad \lim_{r\to\infty} |S_r| = \infty$

similarly, we can show that $\lim_{g\to\infty} |S_g| = \infty$ where z_g is a negative real number for $g > g_0$.

Case (2): Let z_g be a complex number for $g > g_0$. Without loss of generality we may assume a suitable number $e^{i\phi}$ and multiplying by all term by $e^{i\phi}$ in which the axes so that the consider angle ψ is bisected by only real axis and hence obtain x_g which a non-negative real number in which $x_g = R(z_g)$.

If δ_g = amplitude of z_g, then $-\frac{\psi}{2} < \delta_g < \psi_2$, so that, when $g > g_0$,

$$x_g = |z_g| \cos \delta_g > |z_g| \cos \frac{\psi}{2}$$

$\Rightarrow \sum_{\ell=1}^{g} x_\ell > \cos \frac{\psi}{z} \sum_{\ell=1}^{g} |z_\ell| \quad \left(0 < \frac{\psi}{2} < \frac{\pi}{2}\right)$

Thus, $\lim_{g\to\infty} \sum_{\ell=1}^{g} x_g = +\infty$.

By case (1), we may write

$$\sum_{g=1}^{\infty} p_g(w) x_g \to +\infty$$

$\Rightarrow \left|\sum_{g=1}^{\infty} p_g(w) z_g\right| \to +\infty$.

Theorem 9: Each bounded and unbounded terminal point of an infinite series of partial sums $\sum V_g$ is the generalized sum of the series $\sum V_g$for some positive γ -matrix.

Proof of theorem 9: Let $t_g = v_1 + v_2 + \ldots + v_g$

$$= \sum_{\ell=1}^{g} v_g$$

and t is a finite limit point of the partial sum t_g, then there exists a subsequence t_{r_g}which also has a same finite limit t.

i.e. $t_{r_g} \to t$.

If $\overline{\lim_{g\to\infty}}|t_g| = \infty \Rightarrow \overline{\lim_{g\to\infty}}\left|t_{r_g}\right| = \infty$

i.e. $\left|t_{r_g}\right| \to \infty$ but

amplitude t_{r_g} $\boxed{\rightarrow}$ α for some real values of α.

From above two cases we may conclude that there exists γ -matrix $p_{\ell,g} = 1$ for $1 \leq g \leq r_\ell$

and $p_{\ell,g} = 0$ for $g > r_\ell$

where $\ell = 1,2,\ldots$, which transforms $\sum v_g$ into $\sum_{g=1}^{\infty} p_{\ell,g} v_g = t_{r_\ell}$.

Proof of theorem 2:

We define $Q = \left(q_{\ell g}\right)$ be α -matrix corresponding to γmatrix $P = \left(p_{\ell g}\right)$.

Then

$q_{1g} = p_{1g} \qquad (g \geq 1)$

$q_{\ell g} = p_{\ell g} - p_{\ell-1,g} \qquad (\ell > 1, g \geq 1)$

Therefore

$$\lim_{g\to\infty} q_{\ell g} = \lim_{g\to\infty}\left(p_{\ell g} - p_{\ell-1,g}\right)$$

$$= p_\ell - p_{\ell-1}$$

$= q_\ell$ (say)

(12)

where q_ℓ is a finite for all ℓ.

Proof of theorem 3:

We may write

$$|p_{\ell g}| = \left|p_{\ell g} + \sum_{r=2}^{\ell}\left(p_{rg} - p_{r-1,g}\right)\right|$$

$$\leq |p_{\ell g}| + \left|\sum_{r=2}^{\ell}\left(p_{rg} - p_{r-1,g}\right)\right|$$

$$< |p_{\ell g}| + \sum_{r=2}^{\ell} |p_{rg} - p_{r-1,g}|$$

$$< |p_{\ell g}| + M(P)$$

$< N(P)$

(13)

where $N(P)$ is independent of ℓ and g.

Proof of theorem 2:

We have

$r_{\ell g} = \sum_{i=1}^{\infty} p_{\ell i} q_{ig}$

(14)

and

$\lim_{g\to\infty} r_{\ell g} = \lim_{g\to\infty} \left(\sum_{i=1}^{\infty} p_{\ell i} q_{ig} \right)$

(15)

hence by using (13) and [8], we may conclude that

$$|r_{\ell g}| \leq \sum_{i=1}^{\infty} |p_{\ell i}| |q_{ig}|$$

$$< N(P)M(P)$$

$$< N(P)$$

where the constants $N(P)$ is independent of ℓ and g.

Thus by (12) and (14), we may conclude that

$$\lim_{g\to\infty} r \sum_{i=1}^{\infty} \left\{ \lim_{g\to\infty} (p_{\ell i} q_{ig}) \right\}_{\ell g}$$

$\Rightarrow \qquad r_{\ell} = \sum_{i=1}^{\infty} p_{\ell i} q_i$

This concludes the demonstration of the theorem's validity.

In a similar vein, we are able to demonstrate that theorem (4).

Definition 7: A T-matrix $A_z(a_{\ell g})$ is defined to be of type M if (Hill, 1939)

$\sum_{\ell=1}^{\infty}|p_\ell| < \infty$,

and $\sum_{\ell=1}^{\infty} p_\ell a_{\ell g} = 0 \forall g = 1,2,3,\ldots$

$\Rightarrow \quad p_\ell = 0 \forall \ell.$

Definition 8: A γ -matrix is $P = (p_{\ell g})$ is defined to be of the type M if $\sum_{\ell=1}^{\infty}|d_\ell| < \infty$ (15*)

and

$\sum_{\ell=1}^{\infty} d_\ell(p_{\ell g} - p_{\ell-1,g}) = 0$ (16)

$\Rightarrow d_\ell = 0 \forall \ell.$ (17)

Ramanujan (1956) has proved the following theorems:

Theorem A: The product $R = PQ$of a γ -matrix P of type M and an α -matrix Q of type M is a γ -matrix of type M.

Theorem B: If C and D are α -matrices of type M, so is their product matrix E.

Theorem C: If the product $E = CD$ of two α -matrices of type M then C is of type M.

Theorem 10: The product $R = PQ$ of a γ - matrix P of type M and an α -matrix Q of type M is a γ -matrix type M.

Theorem 11: If the product $E = CD$ of type α -matrices C and D be of type M, then C must be of type M.

Theorem 12: The product of two α -matrices of type M is an α -matrix of type M.

Proof of theorem 10:

We have

$(R)_{\ell g} = (PQ)_{\ell g}$,

$r_{\ell g} = \sum_{i=1}^{\infty} p_{\ell i} q_{ig} \qquad (\ell, g \geq 1)$

therefore

$r_{\ell g} - r_{\ell-1,g} = \sum_{i=1}^{\infty}(p_{\ell i} - p_{\ell-1,i})q_{ig}$

(18)

and

$\sum_{\ell=2}^{\infty} a_\ell \left(r_{\ell g} - r_{\ell-1,g}\right) = \sum_{\ell=2}^{\infty} a_\ell \sum_{i=1}^{\infty}(p_{\ell i} - p_{\ell-1,i})q_{ig}$

then by [8] and definition 7, we obtain

$$\sum_{\ell=2}^{\infty} a_\ell \sum_{i=1}^{\infty}(p_{\ell i} - p_{\ell-1,i}) \leq$$

$$\sum_{\ell=2}^{\infty} |a_\ell| \sum_{i=1}^{\infty} |p_{\ell,i} - p_{\ell-1,i}||q_{ig}|$$

$$= \sum_{i=1}^{\infty} |q_{ig}| \sum_{\ell=2}^{\infty} |a_\ell|. |p_{\ell i} - p_{\ell-1,i}|$$

$$< M(P) \sum_{i=1}^{\infty} |q_{ig}| \sum_{\ell=2}^{\infty} |a_\ell|$$

$$< M(P) M_a \sum_{i=1}^{\infty} |q_{ig}|$$

$$< M(P) M_a M(Q)$$

$$< M$$

The right-hand term in Equation (18) can be rearranged to obtain by switching the order of the summations.

$$\sum_{\ell=2}^{\infty} q_\ell (r_{\ell g} - r_{\ell-1,g})$$

$= \sum_{i=1}^{\infty} \left[\sum_{\ell=2}^{\infty} q_\ell \left(p_{\ell i} - p_{\ell-1,i} \right) \right] q_{ig}$

(19)

also, by (6) and given theorem 3, we obtain

$$\sum_{\ell=2}^{\infty}\left|a_\ell\left(p_{\ell i}-p_{\ell-1,i}\right)\right|$$

$$< 2M(P)\sum_{\ell=2}^{\infty}|a_\ell|$$

$< \infty$ (by definition)

Now, if we assume that $\sum_{\ell=2}^{\infty} q_\ell\left(r_{\ell g}-r_{\ell-1,g}\right)=0\forall g$ then because $Q=\left(q_{\ell g}\right)$ is an α -matrix of type M, we get

$\sum_{\ell=2}^{\infty} q_\ell\left(p_{\ell i}-p_{\ell-1,i}\right)=0\forall i$

(20)

$\Rightarrow q_\ell=0$ (by definition)

(21)

$\because$ P is also given to be a γ -matrix of type M.

Combining (18), (19), (20) and (21), we get the required result.

Proof of theorem 11:

For the proof, we assume on the contrary that α -matrix $C=\left(c_{\ell g}\right)$ is not of type M. Then we obtain a sequence $\{a'_\ell\}$ satisfying the conditions

$\sum_{\ell=1}^{\infty}|a'_\ell|<\infty$

(22)

$\sum_{\ell=1}^{\infty} a'_\ell . c_{\ell g}=0$

(23)

but

$a'_\ell\neq 0\forall\ell$

(24)

Since C and D are α -matrices,

$|c_{\ell i}|<N(C) \quad (\ell\geq 1)$

and $\sum_{\ell=}^{\infty}\left|d_{\ell g}\right|<M(D)$ independent of g.

Then by (22)

$\sum_{i=1}^{\infty}\sum_{\ell=1}^{\infty} a'_{\ell} c_{\ell i} d_{ig}$, is absolutely convergent $\forall g$.

Thus by (23),

$$\sum_{\ell=1}^{\infty} a'_{\ell} e_{\ell g} = \sum_{\ell=1}^{\infty} c'_{\ell} \sum_{i=1}^{\infty} c_{\ell i} d_{ig}$$

$$= \sum_{i=1}^{\infty}\left[\sum_{\ell=1}^{\infty} a'_{\ell}.\, c_{\ell i}\right] d_{ig}$$

$= 0$

(25)

since $a'_{\ell} \neq 0 \forall \ell = 1$ and (25) contradicts the hypothesis that $E = CD$ is of type M. Thus, C must be type M.

Proof of theorem 12:

Let P be γ -matrix corresponding to α -matrix C.

If $E = CD$ be the product of two α -matrices C and D then

$e_{\ell g} = \sum_{i=1}^{\infty} c_{\ell i} d_{ig} \qquad (\ell, g \geq 1)$

$$= \sum_{i=1}^{\infty}(p_{\ell i} - p_{\ell-1,i}) d_{ig}$$

$\therefore e_{\ell g} = r_{\ell g} - r_{\ell-1,g}$

(26)

say it follows from theorem (10) the $R = (r_{\ell g})$ is γ -matrix of type M.

Further it obvious, from (26), that R is γ -matrix corresponding to the α -matrix E.

Then by hypothesis, $E = CD$ is α -matrix of type M.

Conclusion

Using absolute permanent matrix transformation, we have proven certain general theorems on the product of matrices and their applicability to infinite series.

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